Study of the rare decay $B \to \mu\nu\gamma$

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Abstract

We present a preliminary study of the rare decay $B \to \mu\nu\gamma$. We define discriminating variables to identify the signal $B \to \mu\nu\gamma$ and reject the physical backgrounds $\Upsilon(4S) \to B^+B^-$, $B^+ \to X, B^- \to Y$ and $\Upsilon(4S) \to B^0\bar{B}^0$, $B^0 \to X, \bar{B}^0 \to Y$. The results of the analysis are discussed aiming at estimating BaBar sensitivity to the signal and the achievable precision of the measurement of the branching ratio.

1 Introduction

The leptonic decays of heavy mesons have both theoretical and experimental interest. The purely leptonic decay $B \to l\nu_l$ can be used to extract the value of $|V_{ub}|$, if the value of f_B, the B decay constant, is known from lattice QCD. Conversely, by measuring $|V_{ub}|$ via semileptonic decays the value f_B can be estimated studying purely leptonic decays.

The difficulty in the experimental observation of the purely leptonic decays of heavy pseudoscalars lies on the effect of helicity suppression which affects these decays: the two back to back decay particles must be in a spin 0 state, so that the lefthanded (righthanded) neutrino (antineutrino) forces the negative (positive) lepton to neutrino's helicity, which introduces a suppression factor of magnitude $(m_l/m_B)^2$ in the decay rate, given by:

$$\Gamma(B \to l\nu) = \frac{G_F^2}{8\pi}|V_{ub}|^2 f_B^2 \left(\frac{m_l}{m_B}\right)^2 m_B^3 \left(1 - \frac{m_l^2}{m_B^2}\right).$$

Using the experimental value of τ_B and $|V_{ub}|$, and the theoretically calculated value of f_B, the Branching Ratio estimates in the case of the muon and electron are:

$$BR(B \to e\nu) \sim 10^{-12} \quad (1)$$
$$BR(B \to \mu\nu) \sim 10^{-7} \quad (2)$$

The mass of the τ lepton is sufficiently high to have a low suppression, so that the expected Branching Ratio is:

$$BR(B \to \tau\nu) \sim 10^{-5}$$

The 10^{-12} BR in the case of the electron case makes the detection of this channel actually impossible, while the hard experimental τ identification, and the low Branching Ratio of the muon channel, make these decays very difficult to observe. Presently only upper limits at 90% C.L. have been found by the CLEO collaboration[1]:

$$BR(B \to e\nu) < 2.1 \times 10^{-5} \quad (3)$$
$$BR(B \to \mu\nu) < 1.5 \times 10^{-5} \quad (4)$$
$$BR(B \to \tau\nu) < 2.3 \times 10^{-5}. \quad (5)$$

In this context, the radiative decay $B \to \mu\nu\gamma$ could play an important role, because of the absence of the helicity suppression effect, which can compensate the presence of the electromagnetic coupling constant α_{EM}. Two types of amplitude contribute to this decay mode: internal bremsstrahlung (IB) photon emission, shown in figure 1, and structure dependent (SD) photon emission, shown in figure 2. While the IB contributions are still helicity suppressed, the SD are not, because they consist of the coupling of the pseudoscalar B meson with the photon in the final state via an intermediate spin 1 axial or vector meson, which subsequently decays in the pair $\mu\nu$. In this case, the final state must be in a spin 1 state, so that the helicity suppression is absent.

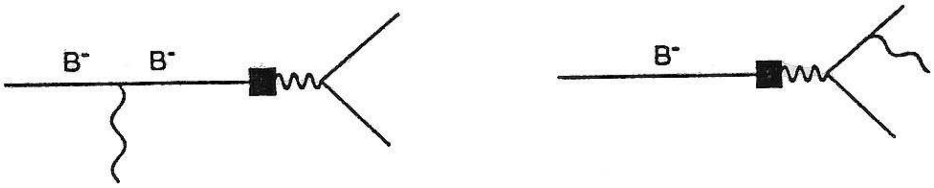

Figure 1: bremsstrahlung diagrams for the decay $B \rightarrow \mu\nu\gamma$.

Figure 2: Structure dependent diagrams for the decay $B \rightarrow \mu\nu\gamma$.

To SD decay amplitudes both vector and axial intermediate meson contribute, and they have different electromagnetic couplings, and decay costants. Theoretical extimates, based on HQET and experimental data of radiative decays of D mesons, show that axial contributions are small compared to vector contributions[2]. Neglecting contributions from intermediate axial mesons, the differential decay rate is[3]:

$$
\begin{aligned}
\frac{d^2\Gamma}{dx\,dy} &= \frac{G_F^2|V_{ub}|^2}{32\pi^2}\alpha m_B^3 f_{B^*}^2 \mu_V^2 \frac{1}{(x+\frac{2\Delta}{m_B})^2} \\
&\quad \times \left[y^2(1-x) + y(3x - x^2 - 2) + \frac{1}{2}(3x^2 - 4x - x^3 + 2)\right],
\end{aligned}
$$

where μ_V is the $BB^*\gamma$ electromagnetic coupling and $\Delta = M_{B^*} - M_B$. In the limit where the b quark mass becomes infinite, the decay constant f_{B^*} is equal to $m_B f_B$ so that also the decay $B \rightarrow \mu\nu\gamma$, can give information on f_B.

Theoretical calculation, based on this argument[2][3] and on QCD potentials [4], give values for the decay branching ratio in the range $0.1 \div 3.6 \times 10^{-6}$, i.e. of the order of magnitude or greater than the purely leptonic one.

Presently, CLEO II collaboration has find the following upper limits[5]:

$$
\begin{aligned}
BR(B \rightarrow \mu\nu\gamma) &< 5.2 \times 10^{-5}\ (90\%CL) \\
BR(B \rightarrow e\nu\gamma) &< 2.0 \times 10^{-4}\ (90\%CL)
\end{aligned}
$$

2 MonteCarlo sample

Our analysis was done with the `Beta` package using `Aslund` fast simulation. We have generated 10^4 signal events, 10^4 events $\Upsilon(4S) \to B^+B^-$, $B^+ \to X$, $B^- \to Y$, and 10^4 events $\Upsilon(4S) \to B^0\bar{B}^0$, $B^0 \to X$, $\bar{B}^0 \to Y$. On this initial sample we found a set of preselections preserving most of the signal, but dramatically reducing the background events. Then, we have generated the final sample of preselected events corresponding to 10^7 background events. We concentrated our analysis in B decays of $\Upsilon(4S)$ neglecting the $q\bar{q}$ background, since the CLEO II collaboration showed that this background can be reduced to much smaller contribution using event shape variables[5], although CLEO II analysis considers muons with momentum above 2.0 GeV.

3 Analysis strategy

The photon and the muon in the final state are direct products of the decay of the B meson, so they can be highly energetic because of the high B meson rest mass (~ 5.28 GeV) as shown in the figures 3 and 4. The same figures also show that most of the background photon are distribuited at momenta smaller than 500 MeV. The selection of photons with a momentum greater than 500 MeV greatly reduces the background with low signal loss.

Since most of the signal muon momentum is greater than 500 MeV, we also select events with muon momentum greater than 500 MeV.

The available phase space boundaries density of the $B \to \mu\nu\gamma$ decay are given by the relations:

$$\begin{aligned} p_\mu &\leq \frac{M_B}{2} \\ p_\gamma &\leq \frac{M_B}{2} \\ p_\mu + p_\gamma &\geq \frac{M_B}{2} \end{aligned}$$

For high momentum muons and photons, it is a good approximation to consider the center of the mass reference frame coincident with the B at rest reference frame, because of the low B momentum (~ 360 MeV) compared to the muon and photon momenta. The distributions of the muon versus photon momenta are shown in figure 5. The sum of the muon and photon momenta is a good variable that select the signal and reject most of the background, as shown in figure 6. In order to reject background photons coming from π^0 decay, we recontruct the invariant mass of two photon (figure 7) in the following way: fixed the signal photon candidate, we loop on the other photons in the event and we take the pair with the invariant mass nearest the π^0 mass; if the this invariant mass is in the range $105 \text{ MeV} < m_{\gamma\gamma} < 165 \text{ MeV}$, we reject the signal photon candidate.

In the approximation that the B decays at rest in the CM reference frame, the neutrino momentum can be reconstructed from the measured photon and muon momenta using energy-momentum conservation. We can recostruct the B invariant mass and determine the missing momentum excluding the reconstructed neutrino $\vec{p}'_{Miss}$. Figure

8 shows the reconstructed B mass and figures 9 and 10 show $\vec{p}'_{Miss}$ and transverse $\vec{p}'_{Miss}$ distributions respectively.

In the laboratory reference frame, the B meson maximum trasverse momentum (~ 300 MeV) can be neglected compared the momentum along z axis (~ 3 GeV) that the B meson receives from the boost of the $\Upsilon(4S)$. If $\vec{p}_\mu$ is the muon momentum, $\vec{p}_\gamma$ is the photon momentum and $\vec{p}_B$ is the B momentum, in this approximation, the angle between $\vec{p}_\mu + \vec{p}_\gamma$ and z axis (θ), and the angle between $\vec{p}_\mu + \vec{p}_\gamma$ and $\vec{p}_B$ (θ') coincide.

Using the kinematical relation that holds between the momenta of the muon, the photon, the B and $\cos\theta'$ we have:

$$\cos\theta' = \frac{p_B^2 + |\vec{p}_\mu + \vec{p}_\gamma|^2 - (E_B - E_\mu - E_\gamma)^2}{2p_B|\vec{p}_\mu + \vec{p}_\gamma|};$$

we can define the selection variable:

$$D = \cos\theta - \frac{p_B^2 + |\vec{p}_\mu + \vec{p}_\gamma|^2(E_B - E_\mu - E_\gamma)^2}{2p_B|\vec{p}_\mu + \vec{p}_\gamma|},$$

which must be approximatively zero for the signal. In figures 11 and 12 the distributions of the variable D in two different ranges are shown.

4 Selection criteria and efficiencies

We have tested several combinations of selections on the discriminating variables previously described, and we have choosed the one which presented the highest signal to background ratio. The selection criteria are listed below:

- Muon momentum $p_\mu > 500$ MeV.
- Photon momentum $p_\gamma > 500$ MeV.
- Phase spaces consistency: $p_\mu + p_\gamma > 2.5$ GeV.
- π^0 rejection: 105 MeV $< m_{\gamma\gamma} < 165$ MeV.
- Reconstructed B Mass: 5.0 GeV $< M_B <$ 5.6 GeV.
- $-0.1 < D < 0.1$.
- $p'_{Miss} < 1$ GeV.
- $p'_{MissT} < 1$ GeV.

The signal and background efficiencies with this selections are the following:

signal	0.150 ± 0.004
background	$(4 \pm 2) \times 10^{-7}$

5 Statistical analysis

Let b be the average number of expected background events; the probability to observe n_b background events follows the Poisson distribution. Let s be the average number of expected signal events:

$$s = 2\epsilon_s n_{B^+B^-}\ BR$$

where ϵ_s is the signal efficiency, $n_{B^+B^-}$ is the number of observed B^+B^- events, BR is the signal Branching Ratio, and the factor 2 arise from observing both signal positive and negative charge modes.

If n is the number of observed events, an estimate of s is $n-b$, and this number has to be compared to the statistical fluctuation of background events $\sigma_b = \sqrt{b}$. Figure 13 shows the significance:

$$S = \frac{s}{\sqrt{b}}$$

with a sample of 10^7 and $10^8 B\bar{B}$ collected events.

The probability α that all the $s+b$ events are due only to background statistical fluctuation in the gaussian approximation is:

$$\begin{aligned}\alpha &= \frac{1}{\sqrt{2\pi b}}\int_{s+b}^{\infty} e^{\frac{-(x-b)^2}{2b}}dx \\ &= \frac{1}{\sqrt{2\pi}}\int_{S}^{\infty} e^{-x^2/2}dx.\end{aligned}$$

Figure 14 shows the probability α as a function of the BR, with a sample of 10^7 and $10^8 B\bar{B}$ collected events. If we define, by convention, a statistically meaningful signal when it corresponds to a probability of background fluctuation less than 10^{-3}, from figure 14 we determine that BABAR will be sensitive to a $BR > 2\times10^{-6}$ with a sample of $10^7 B\bar{B}$ collected events, and to $BR > 5\times10^{-7}$ with a sample of $10^8 B\bar{B}$ collected events.

In presence of a statistically meaningful signal, the BR can be determined to be:

$$BR = \frac{n-b}{2n_{B^+B^-\epsilon_s}}.$$

If the errors on the exstimate of b and $n_{B^+B^-}$ are neglegible compared with the fluctuation of the number of observed events $\sqrt{s+b}$, the relative error on the measured BR is:

$$\frac{\Delta BR}{BR} = \frac{\sqrt{s+b}}{s}.$$

This estimate of the Branching Ratio relative error as a function of the branching ratio is reported in figure 15.

6 Conclusions

We presented a preliminary study on the decay $B\rightarrow\mu\nu\gamma$. The analysis strategy and the selection criteria we developed lead to a signal efficiency $\epsilon_s = 0.150\pm0.004$, and a residual background efficiency $\epsilon_b = (4\pm2)\times10^{-7}$. Our preliminary study shows that, with the considered selection criteria, BABAR experiment will be able to detect a statistically meaningful signal of $B\rightarrow\mu\nu\gamma$ for a Branching Ratio BR in the range:

- $1 \div 2 \times 10^{-6}$, after 10^7 events have been collected
- $4 \div 6 \times 10^{-7}$, after 10^8 events have been collected.

The relative error of the signal BR measure is in the range $10\% \div 25\%$ when the BR is in the range $1 \div 3 \times 10^{-6}$ with 10^8 events collected.

7 Acknowlegments

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References

[1] CLEO collaboration, M. Artuso et al., Phys. Rev. Lett. 75 (1995) 785.

[2] P.Colangelo et al., Phys. Lett. B 372 (1996) 331.

[3] G. Burdman et al., Phys. Rev. D 51 (1995) 111.

[4] D. Atwood et al., BNL-60950
P. Colangelo et al., hep-ph/9606219 (1996) preprint.

[5] T.E. Browder et al., Phys. Rev. D 56 (1997) 11.

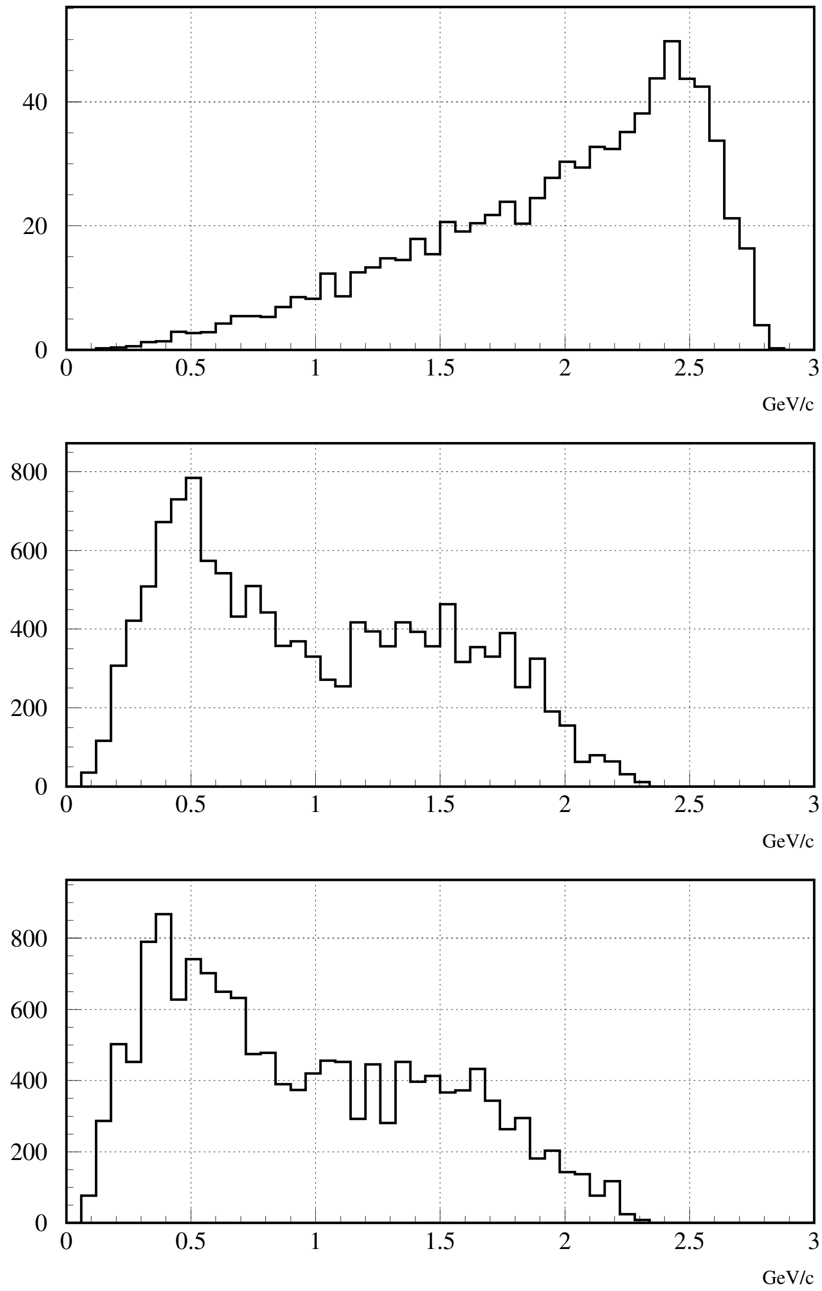


Figure 3: *Muon momentum spectrum for signal (top) and backgrounds* $B^0\bar{B}^0$ *(middle) and* B^+B^- *(bottom).*

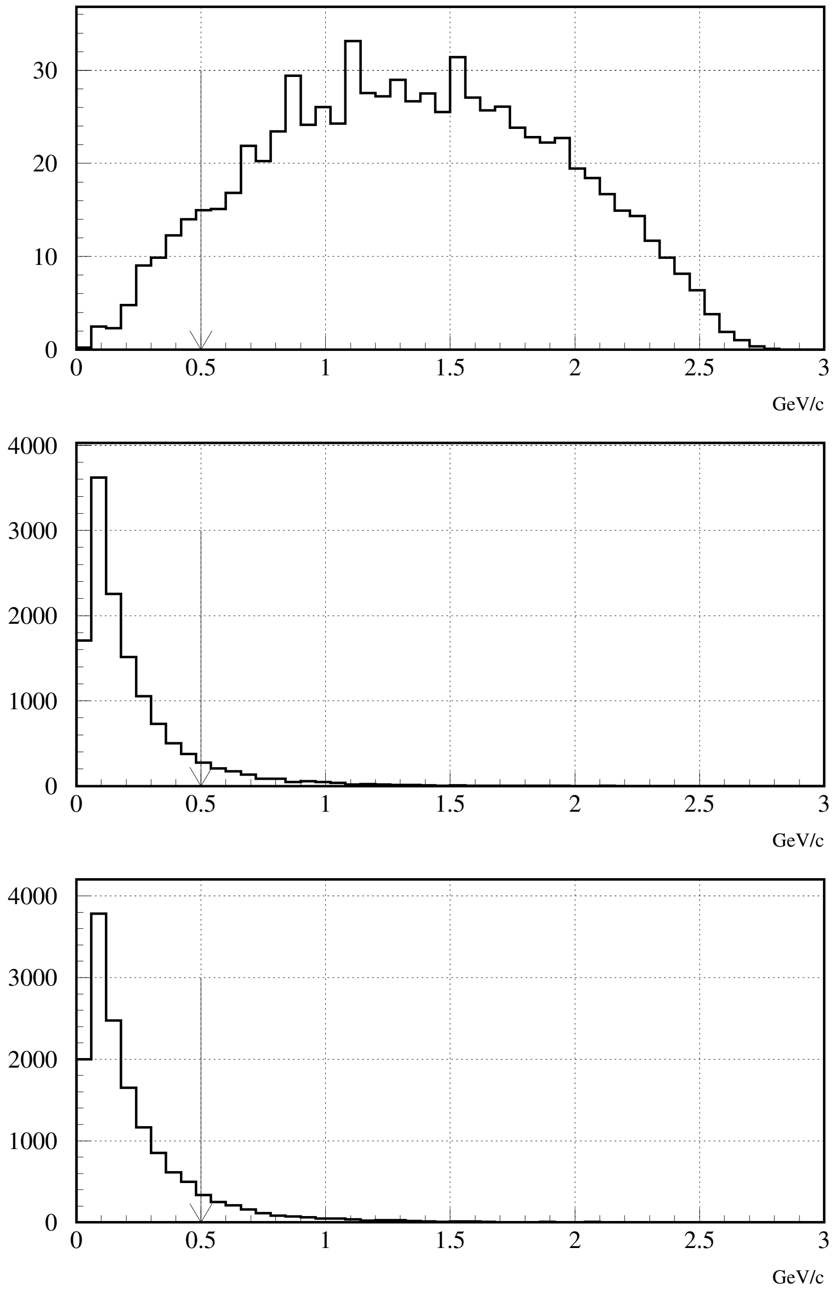


Figure 4: *Photon momentum spectrum for signal and background.*

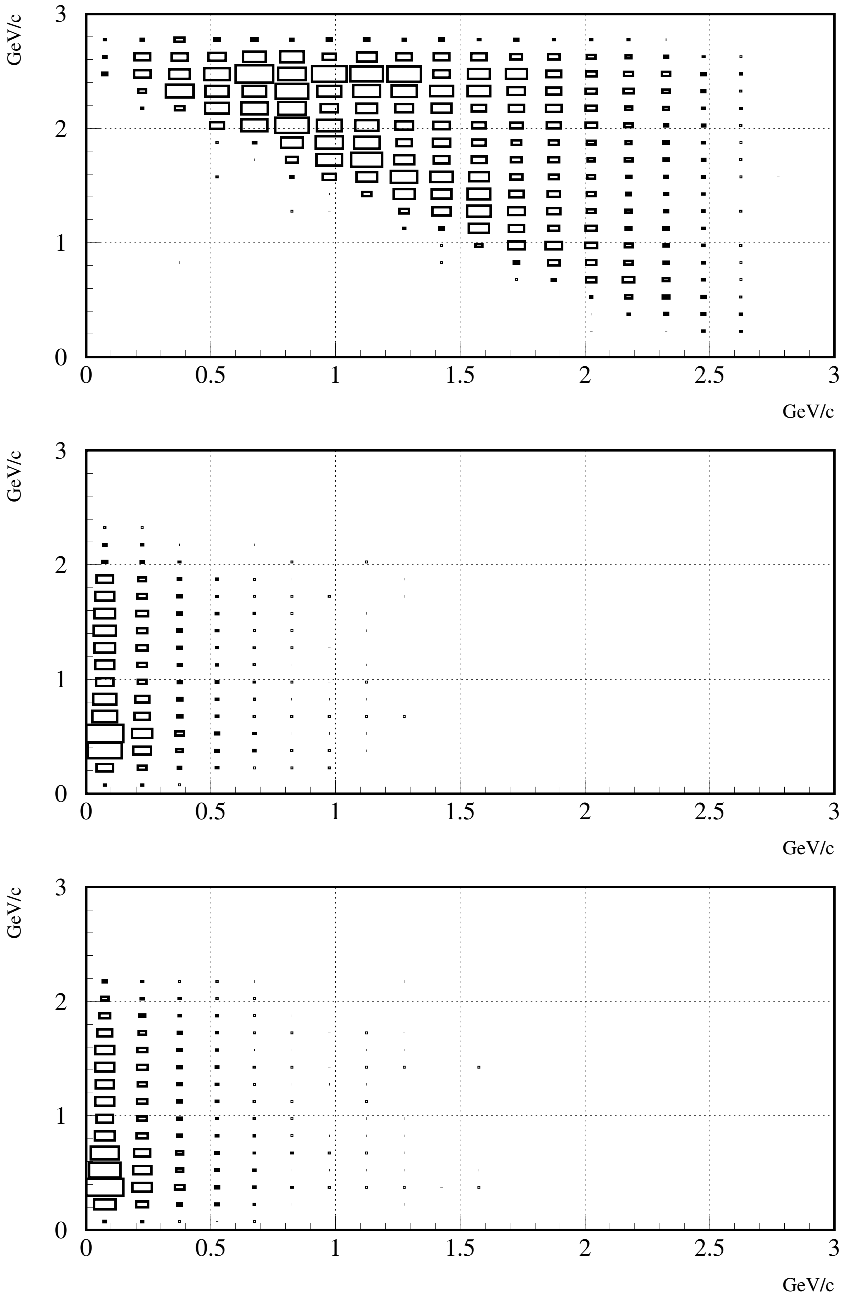


Figure 5: *Muon versus Photon momentum spectrum for signal and background.*

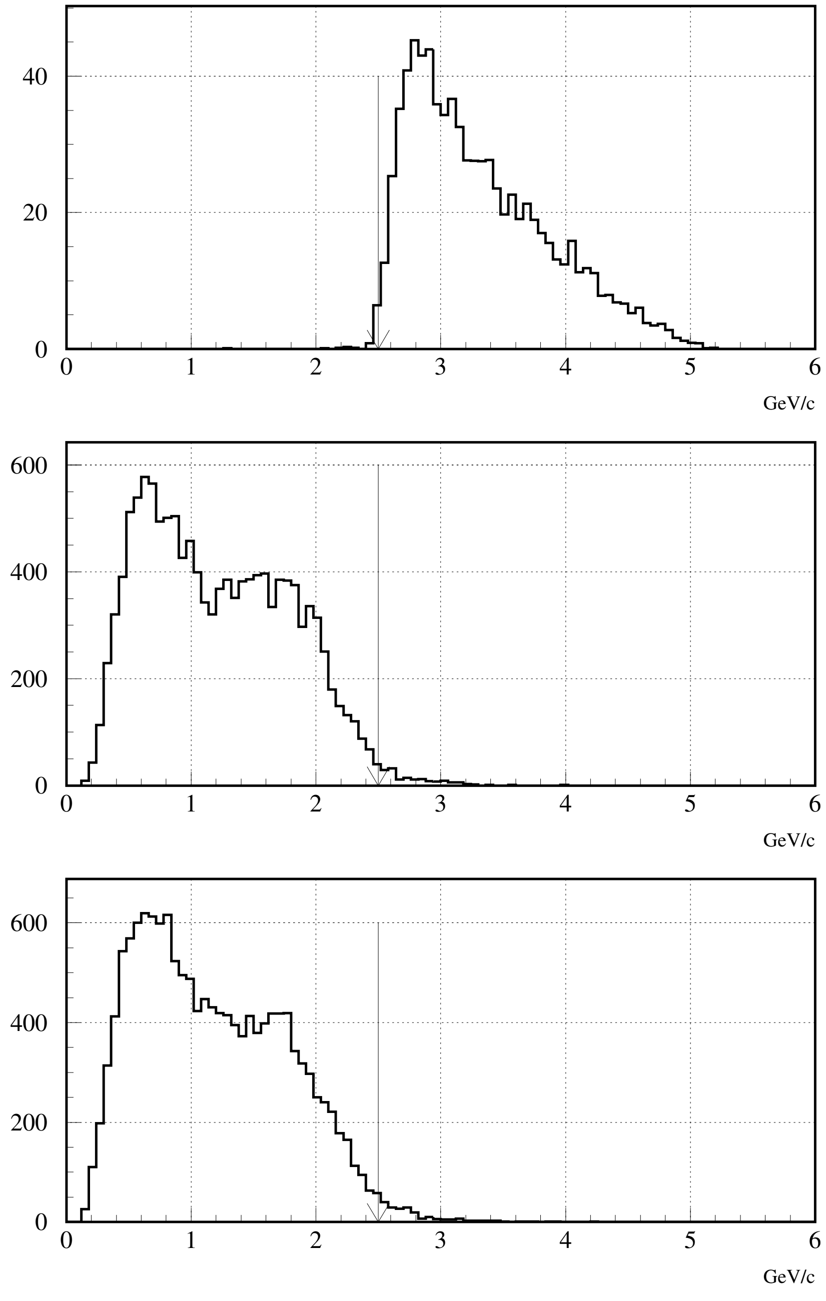


Figure 6: $p_\mu + p_\gamma$ *for signal and background.*

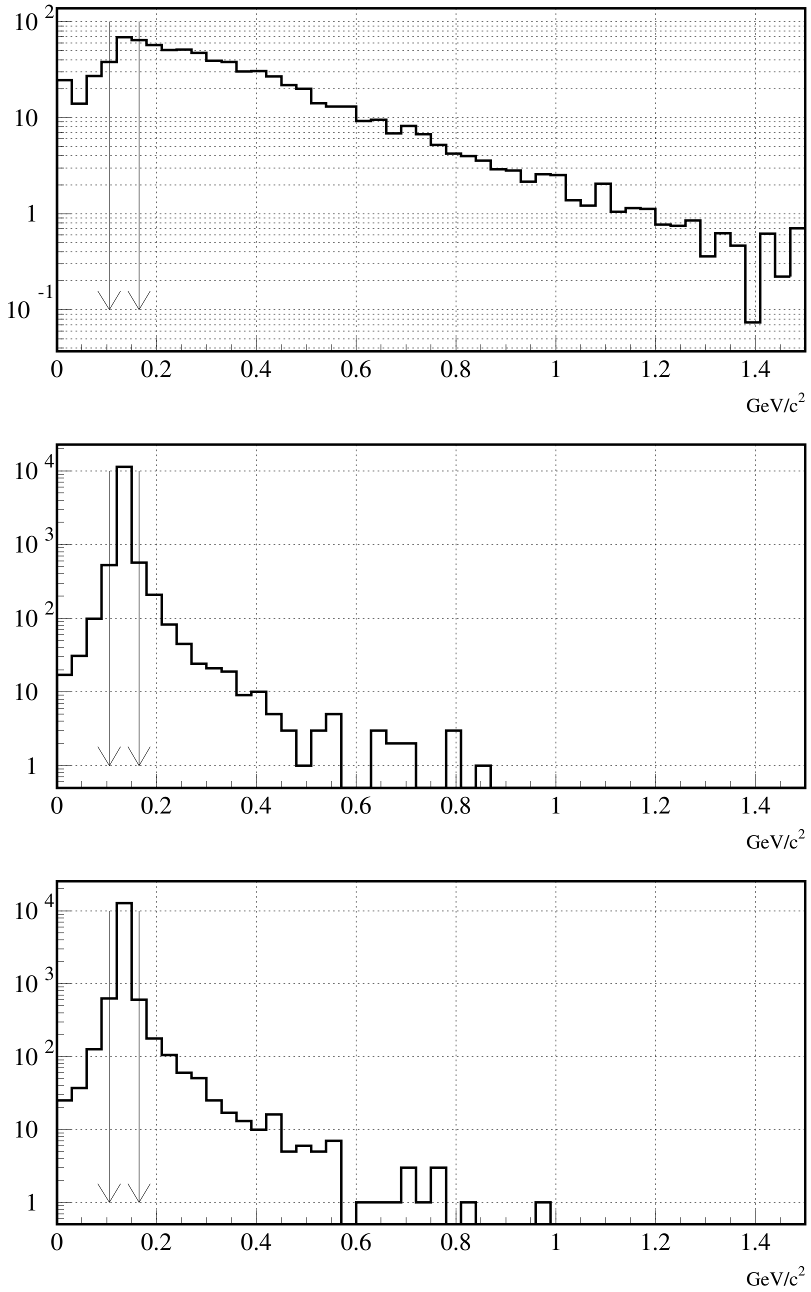


Figure 7: π^0 *mass for signal and background.*

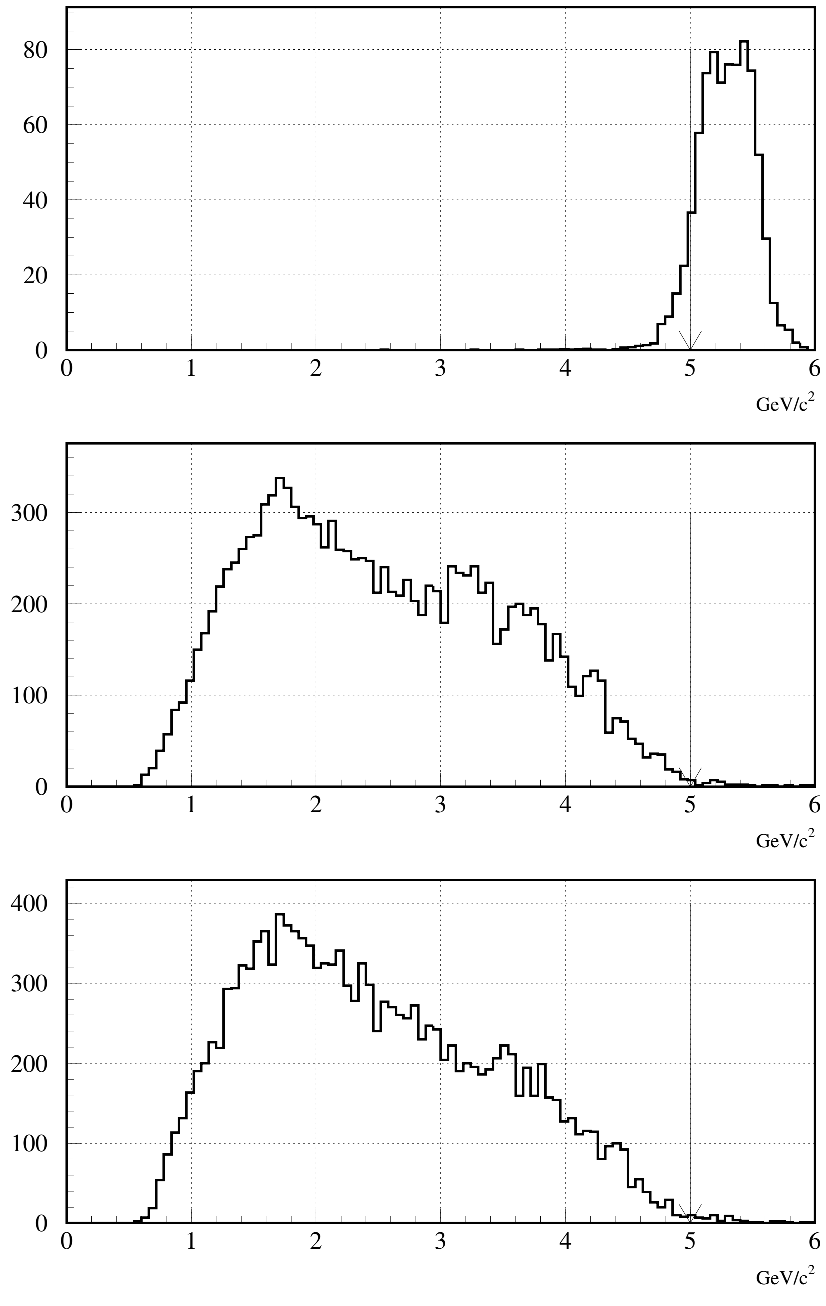


Figure 8: *Reconstructed B Mass for signal and background.*

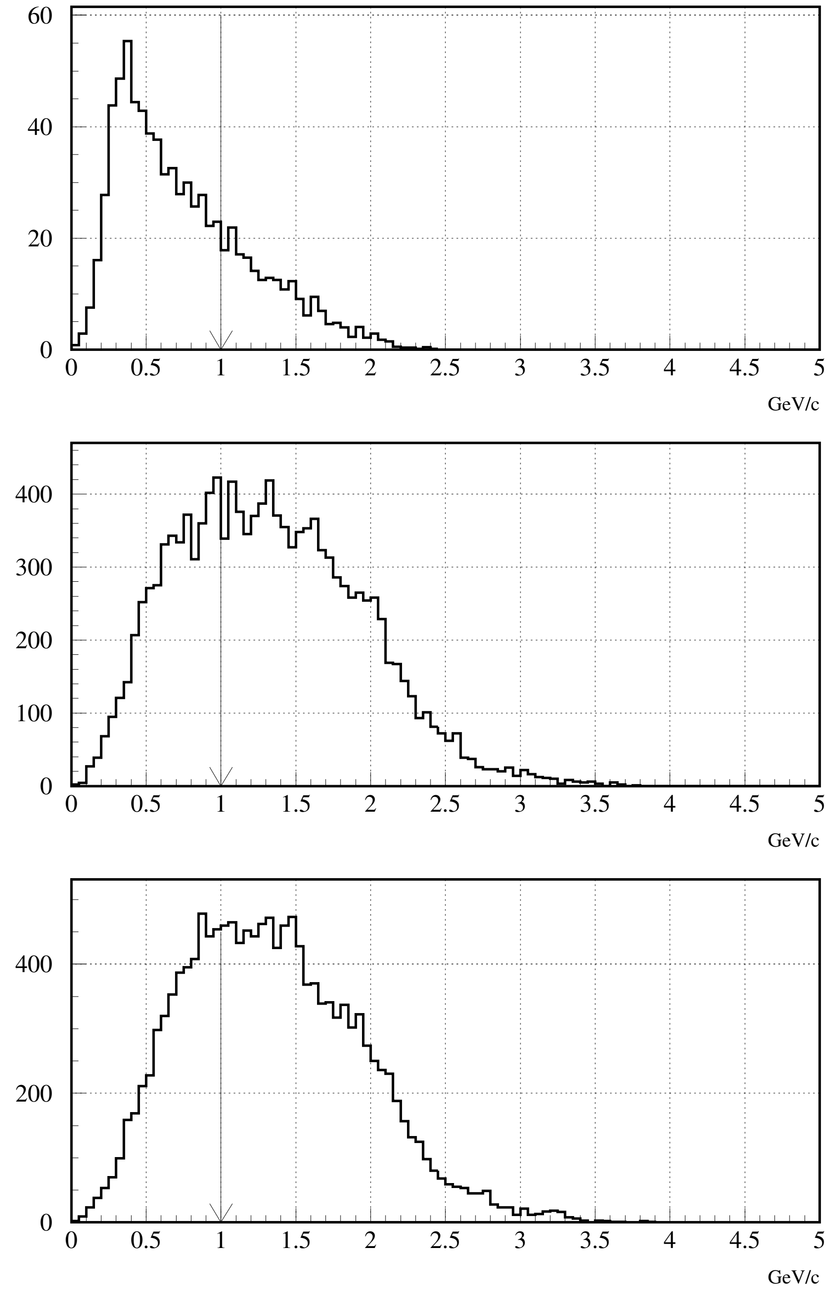


Figure 9: *Corrected missing momentum for signal and background.*

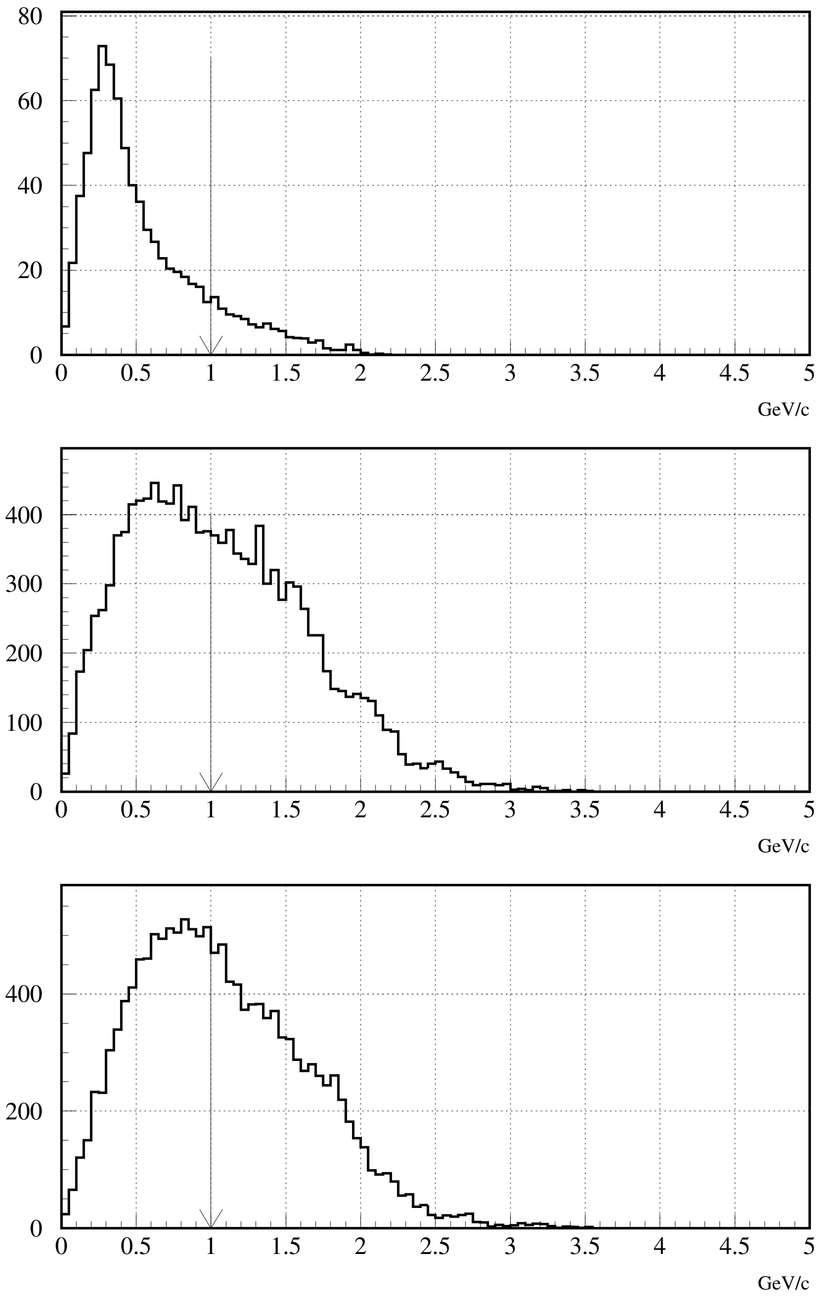


Figure 10: *Corrected missing transverse momentum for signal and background.*

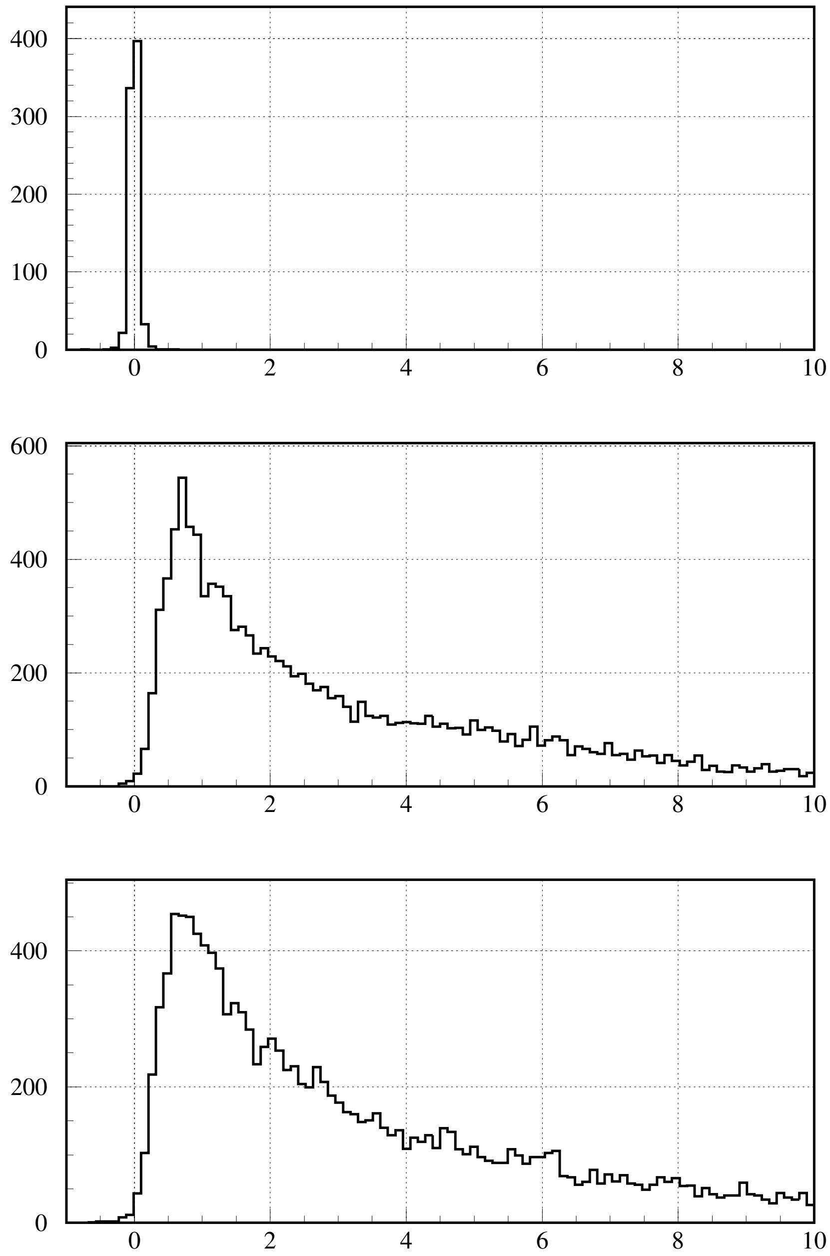


Figure 11: *Difference variable for signal and background.*

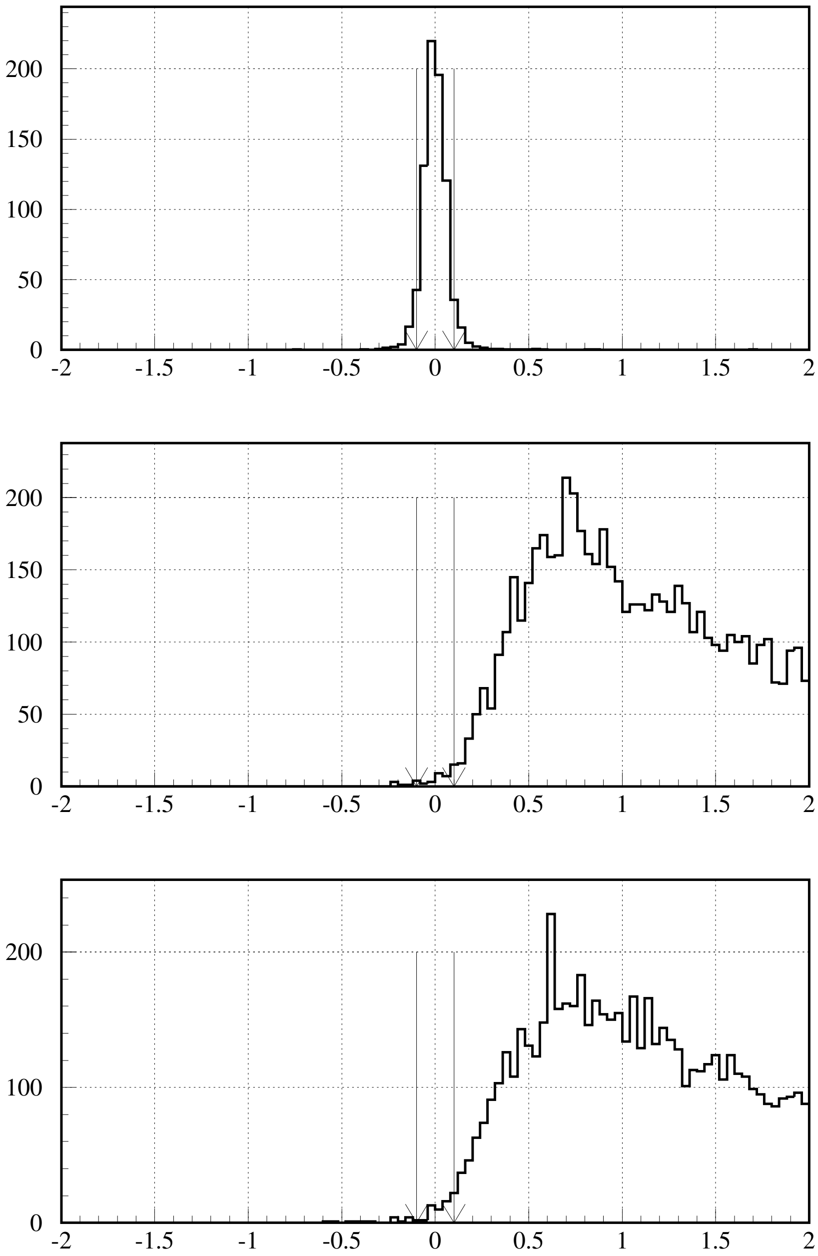


Figure 12: *Difference variable in the region* $-0.1 < D < 0.1$ *for signal and background.*

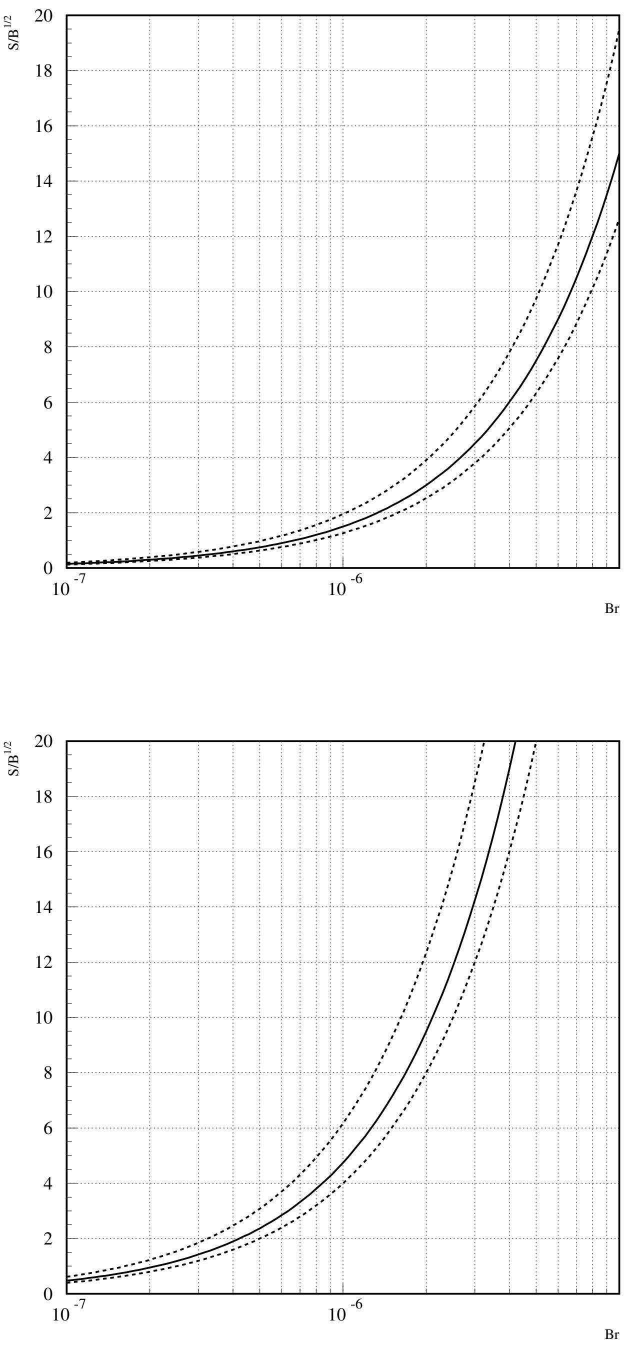
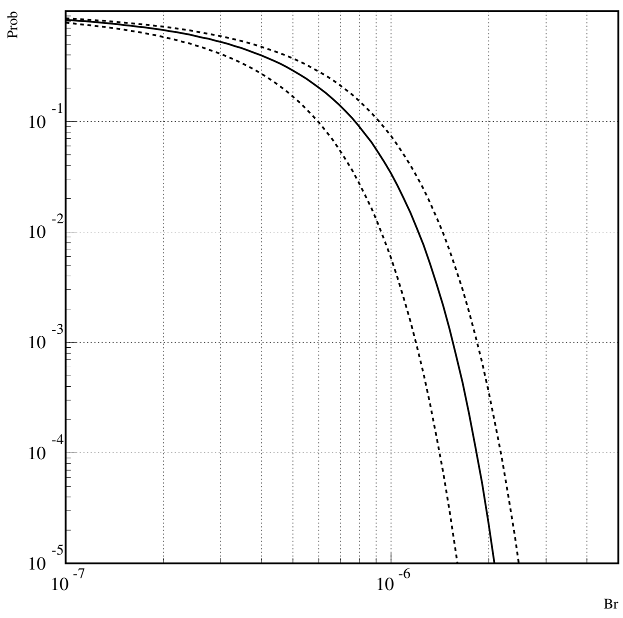


Figure 13: *Significance after* 10^7 *and after* $10^8 B\bar{B}$ *events have been collected*



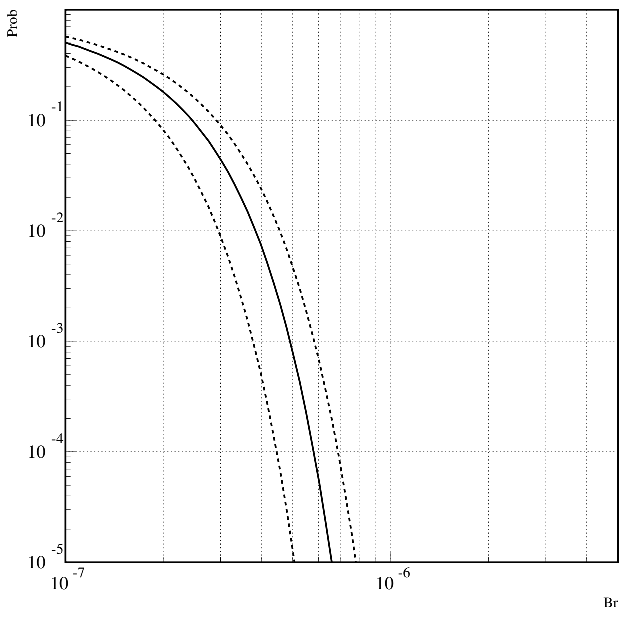


Figure 14: *Probability* α *of a background fluctuation equal to expected signal after* 10^7 *and after* $10^8 B\bar{B}$ *events have been collected*

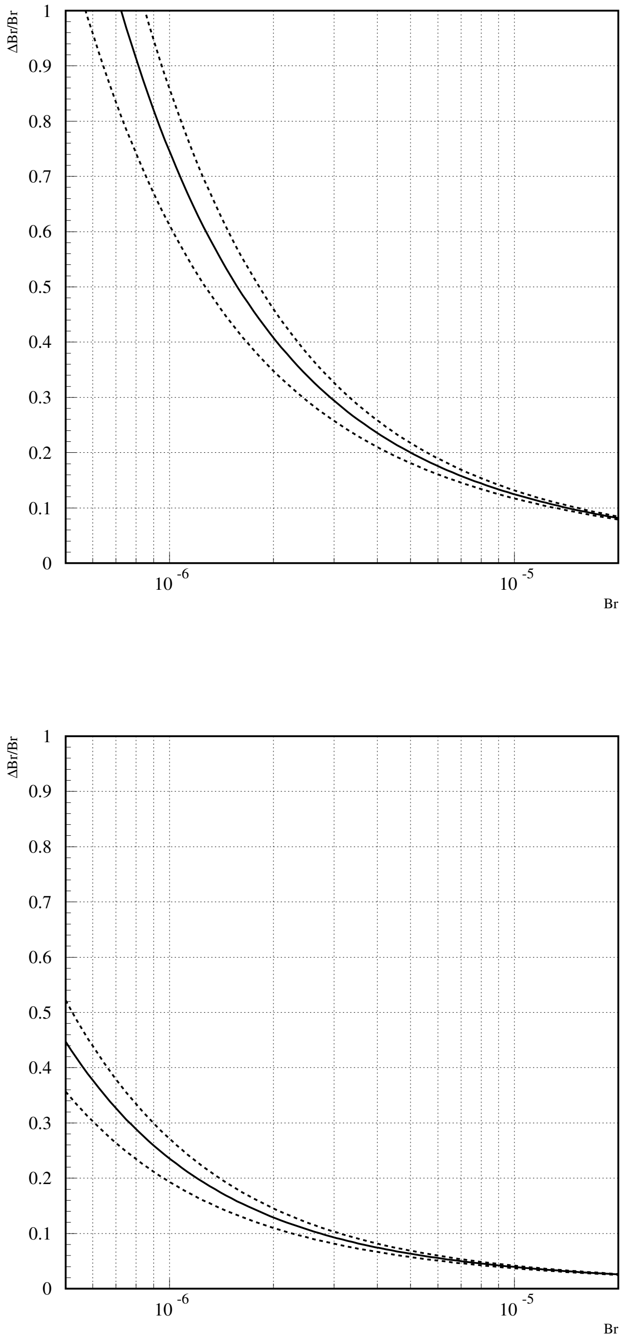


Figure 15: *Relative error $\Delta BR/BR$ after 10^7 and after $10^8 B\bar{B}$ events have been collected*