The MSW effect and Neutrino Astrophysics

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* Context
* Refraction, Resonance, Adiabaticity
* MSW: physical picture of the effect
* Large mixing MSW solution of the solar neutrino problem
* Supernova neutrinos and MSW effect

Context:

Neutrino mixing and Oscillations

B. Pontecorvo,
Z. Maki, M. Nakagawa, S. Sakata

Spectroscopy of Solar Neutrinos

J.N.Bahcall
G.T.Zatsepin, V.A. Kuzmin

Neutrino mass

W. Pauli
E. Fermi

Matter effect Neutrino Refraction

L.Wolfenstein

Homestake Experiment

R. Davis Jr.,
D.S. Hammer,
K.S. Hoffman

References

[1] L. Wolfenstein, ``Neutrino oscillations in matter'', Phys. Rev. D17, (1978) 2369-2374.

[2] L. Wolfenstein, ``Effect of matter on neutrino oscillations'', In Proc. of ``Neutrino -78'', Purdue Univ. C3 - C6.

[3] L. Wolfenstein, ``Neutrino oscillations and stellar collapse'', Phys. Rev. D20, (1979) 2634 - 2635.

[4] S. P. Mikheyev and A. Yu. Smirnov, ``Resonance enhancement of oscillations in matter and solar neutrino spectroscopy'', Sov. J. Nucl. Phys. 42 (1985) 913 - 917.

[5] S. P. Mikheyev and A. Yu. Smirnov, ``Resonance amplifications of ν– oscillations in matter and solar neutrino spectroscopy'', Nuovo Cimento C9 (1986) 24.

[6] S. P. Mikheyev and A. Yu. Smirnov, ``Neutrino oscillations in variable-density medium and ν-bursts due to gravitational collapse of stars'', Sov. Phys. JETP, 64 (1986) 4 - 7.

[7] S. P. Mikheyev and A. Yu. Smirnov, Proc. of the 6th Moriond workshop on ``Massive neutrinos in astrophysics and particle physics, Tignes, France, eds. O Fackler and J. Tran Thanh Van, (1986) p.355.

Flavors, masses, mixing

Flavor neutrino states:

ν_e ν_μ ν_τ

e μ τ

correspond to certain charged leptons

interact in pairs

Eigenstates of the CC weak interactions

ν_s Sterile neutrinos?

Mass eigenstates

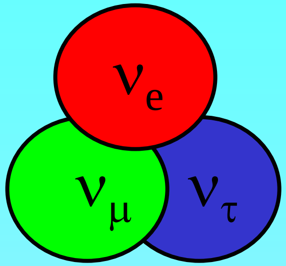
ν_1 ν_2 ν_3

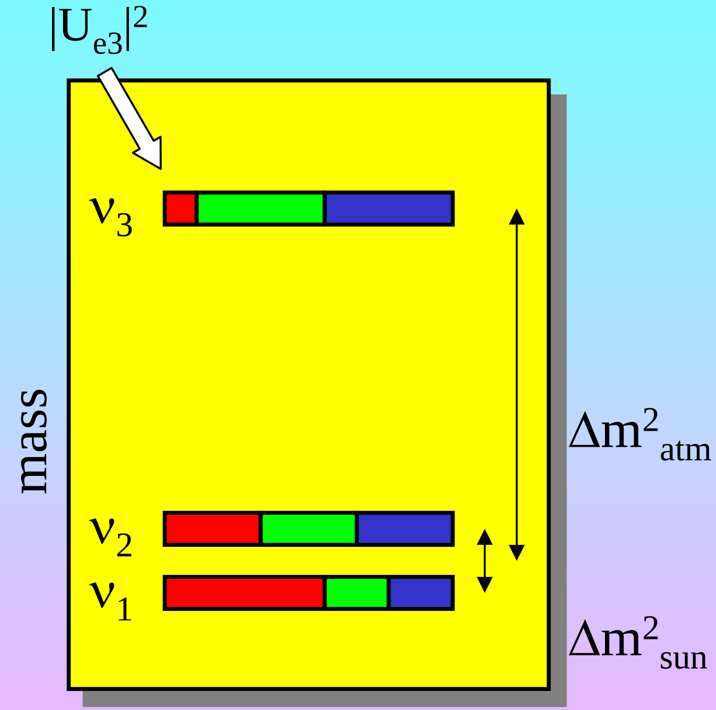
m_1 m_2 m_3

Mixing

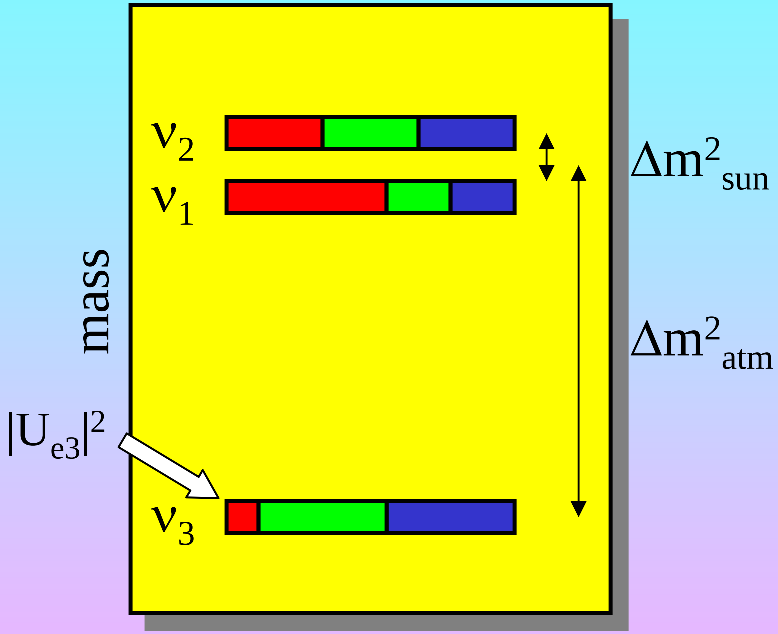
Flavor states $\neq$ Mass eigenstates

Mass spectrum and mixing





Normal mass hierarchy (ordering)



Inverted mass hierarchy (ordering)

- Type of mass spectrum: with Hierarchy, Ordering, Degeneracy ⇒ absolute mass scale
- Type of the mass hierarchy: Normal, Inverted
- U_{e3} = ?

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Two aspects of mixing

vacuum mixing angle

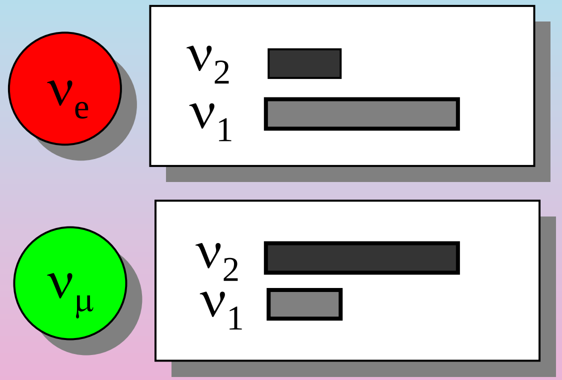
$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

inversely

$$\nu_2 = \sin\theta\, \nu_e + \cos\theta\, \nu_\mu$$
$$\nu_1 = \cos\theta\, \nu_e - \sin\theta\, \nu_\mu$$

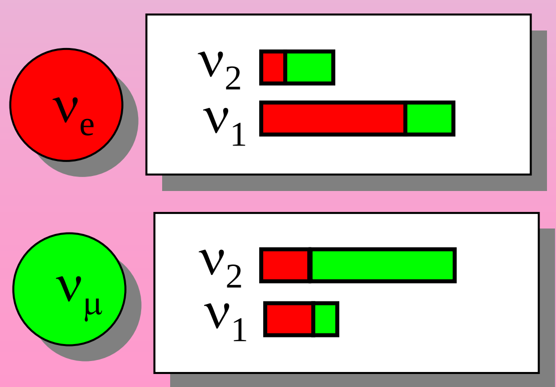
coherent mixtures of mass eigenstates

flavor composition of the mass eigenstates



wave packets

Flavors of eigenstates



The relative phases of the mass states in ν_e and ν_μ are opposite

Interference of the parts of wave packets with the same flavor depends on the phase difference $\Delta\phi$ between ν_1 and ν_2

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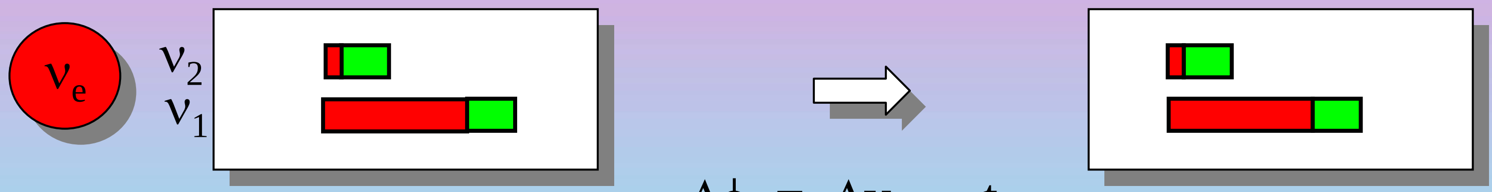
Vacuum oscillations

Propagation in vacuum:

- Flavors of mass eigenstates do not change
- Admixtures of mass eigenstates do not change: no ν_1 <–> ν_2 transitions

Determined by θ



$\Delta\phi = 0$

$\Delta\phi = \Delta v_{phase}\, t$

- Due to difference of masses ν_1 and ν_2 have different phase velocities:

$$\Delta v_{phase} = \frac{\Delta m^2}{2E}$$

$\Delta m^2 = m_2^2 - m_1^2$

oscillations:

effects of the phase difference
increase which changes
the interference pattern

Oscillation length:

$$l_\nu = 2\pi/\Delta v_{phase} = 4\pi E/\Delta m^2$$

Amplitude (depth) of oscillations:

$$A = \sin^2 2\theta$$

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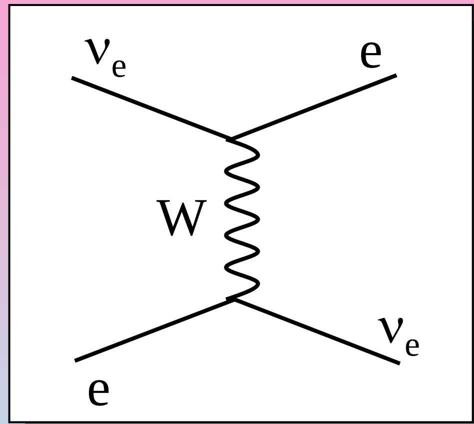
Matter Effect: Refraction

L. Wolfenstein, 1978

Elastic forward scattering → Potentials V_e, V_μ

- $V \sim 10^{-13}$ eV inside the Earth for E = 10 MeV
- Difference of potentials is important → for ν_e ν_μ :



$$V_e - V_\mu = \sqrt{2}\, G_F n_e$$

- Refraction index:

$$n - 1 = V / p$$

- Refraction length:

$$l_0 = 2\pi / (V_e - V_\mu) = \sqrt{2}\, \pi / G_F n_e$$

- n - 1
  - $\sim 10^{-20}$ inside the Earth
  - $< 10^{-18}$ inside the Sun
  - $\sim 10^{-6}$ inside the neutron star

- Neutrino optics → focusing of neutrinos fluxes by stars
  complete internal reflection, etc

Neutrino eigenstates in matter

	in vacuum:		in matter:
Effective Hamiltonean	H_0	⇨	$H = H_0 + V$
Eigenstates	ν_1, ν_2	⇨	ν_{1m}, ν_{2m}
Eigenvalues	m_1, m_2 $m_1^2/2E, m_2^2/2E$	⇨	m_{1m}, m_{2m} H_{1m}, H_{2m}

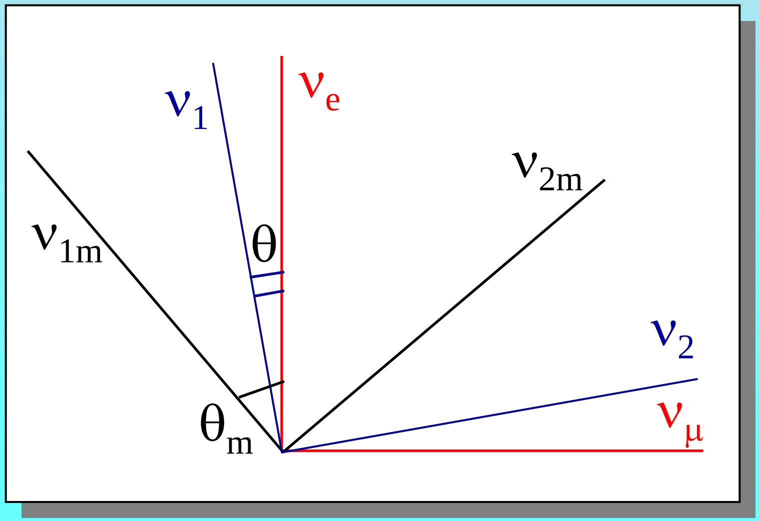
$V = V_e - V_\mu$

depend on n_e, E

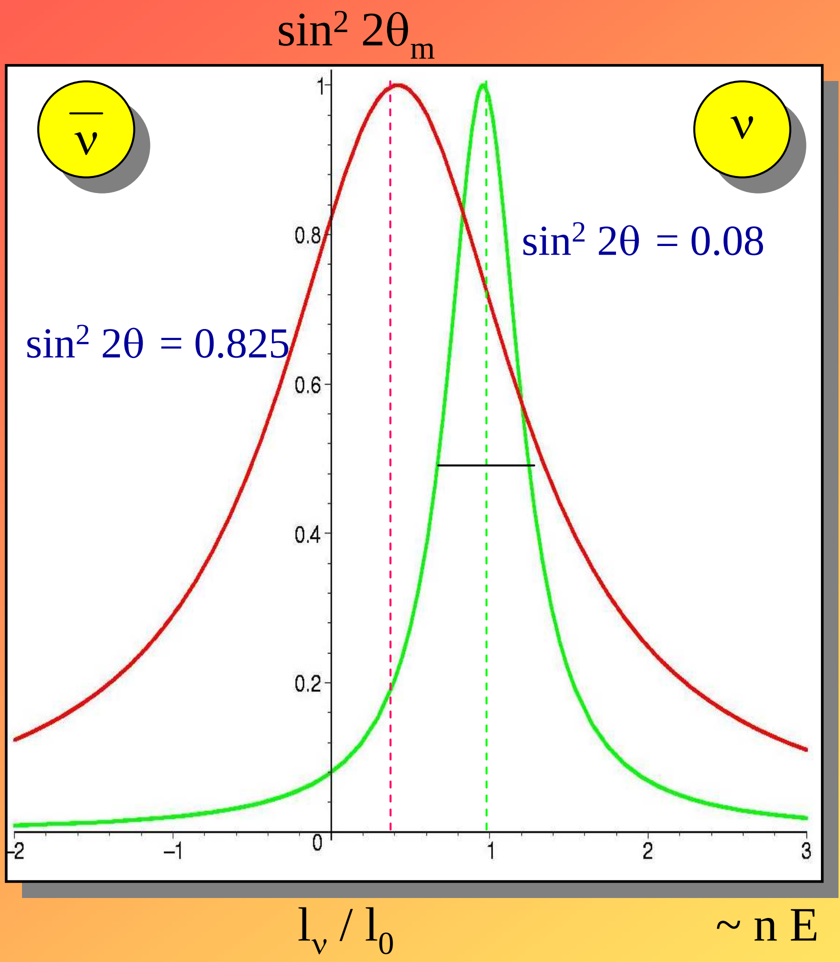
Mixing in matter

is determined with respect to eigenstates in matter

θ_m is the mixing angle in matter



Resonance



- Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$
- Resonance layer: $n = n_R \pm \Delta n_R$

In resonance:

$$\sin^2 2\theta_m = 1$$

- Mixing in matter is maximal
- Level split is minimal

$$l_\nu = l_0 \cos 2\theta$$

Vacuum oscillation length $\approx$ Refraction length

For large mixing: $\cos 2\theta = 0.4 - 0.5$
the equality is broken
the case of strongly coupled system
➡ shift of frequencies

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Level crossing

Dependence of the neutrino eigenvalues on the matter potential (density)

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

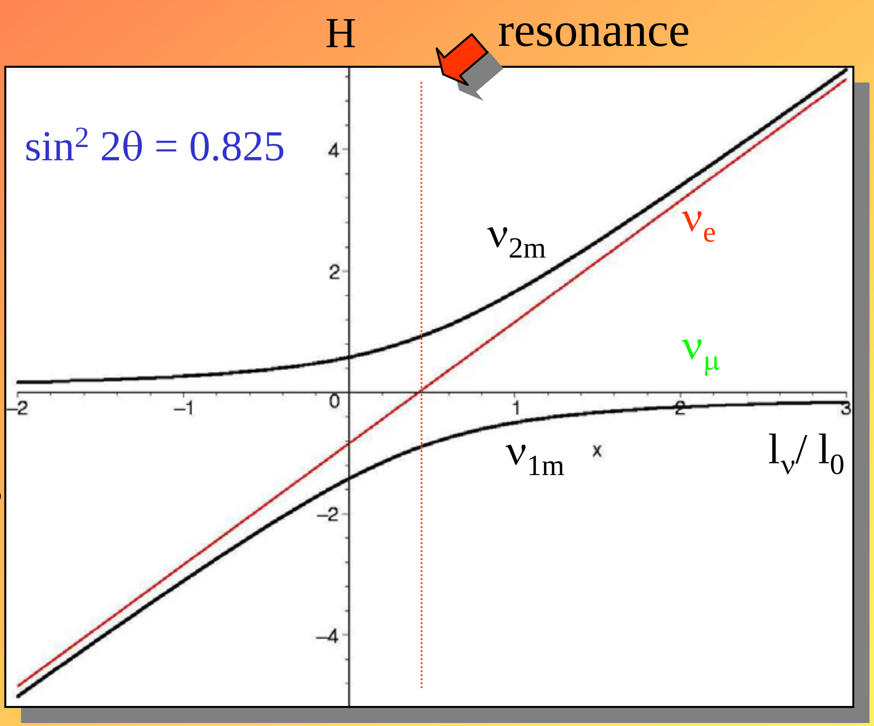
$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance

- the level split in minimal
- the oscillation length is maximal

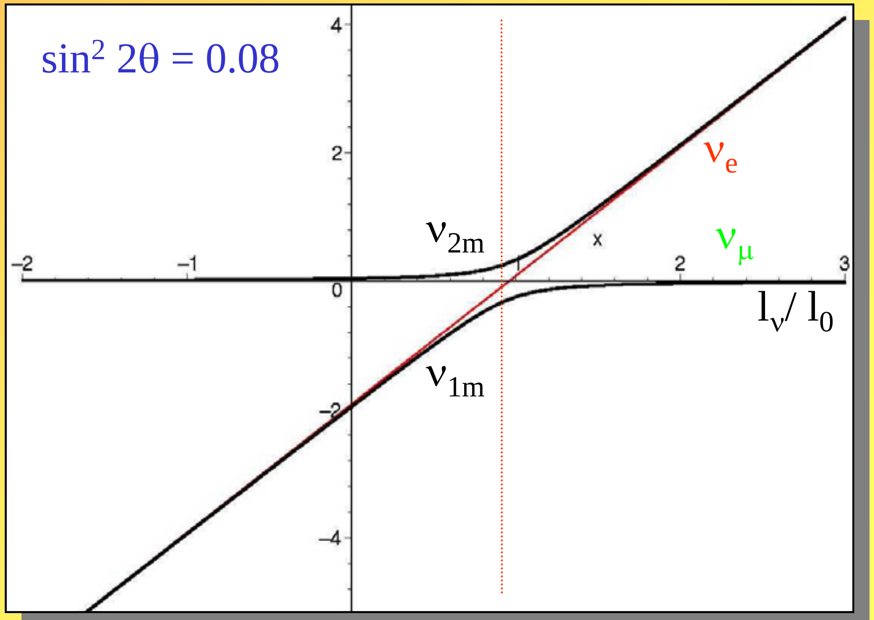
For maximal mixing: at zero density

Large mixing



ν

Small mixing



Two effects

Resonance enhancement of neutrino oscillations

Adiabatic (partially adiabatic) neutrino conversion

	Resonance enhancement	Adiabatic conversion
Density profiles:	Constant density	Variable density
Degrees of freedom:	Change of the phase difference between neutrino eigenstates	Change of mixing, or flavor of the neutrino eigenstates

In general: *Interplay of oscillations and adiabatic conversion*

MSW

Oscillations in matter

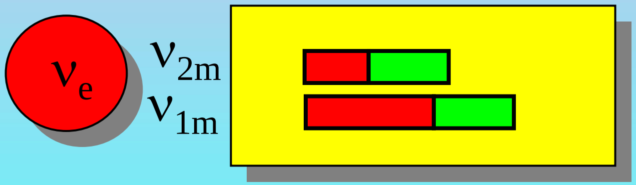
In uniform matter (constant density)
mixing is constant

$\theta_m(E, n)$ = constant

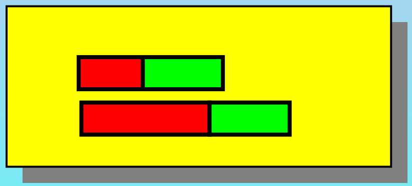
- Flavors of the eigenstates do not change
- Admixtures of matter eigenstates do not change: no $\nu_{1m} <-> \nu_{2m}$ transitions
- Monotonous increase of the phase difference between the eigenstates $\Delta\phi_m$

→ **Oscillations**

as in vacuum



$\Delta\phi_m = 0$

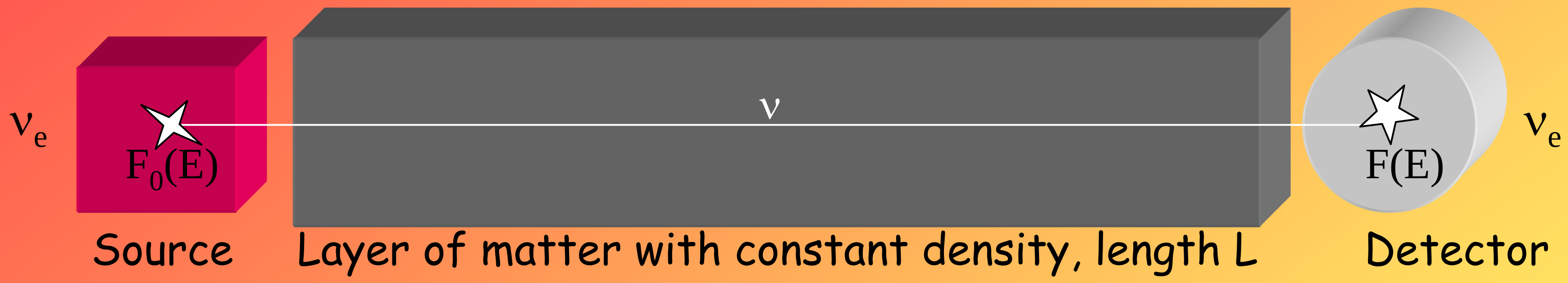


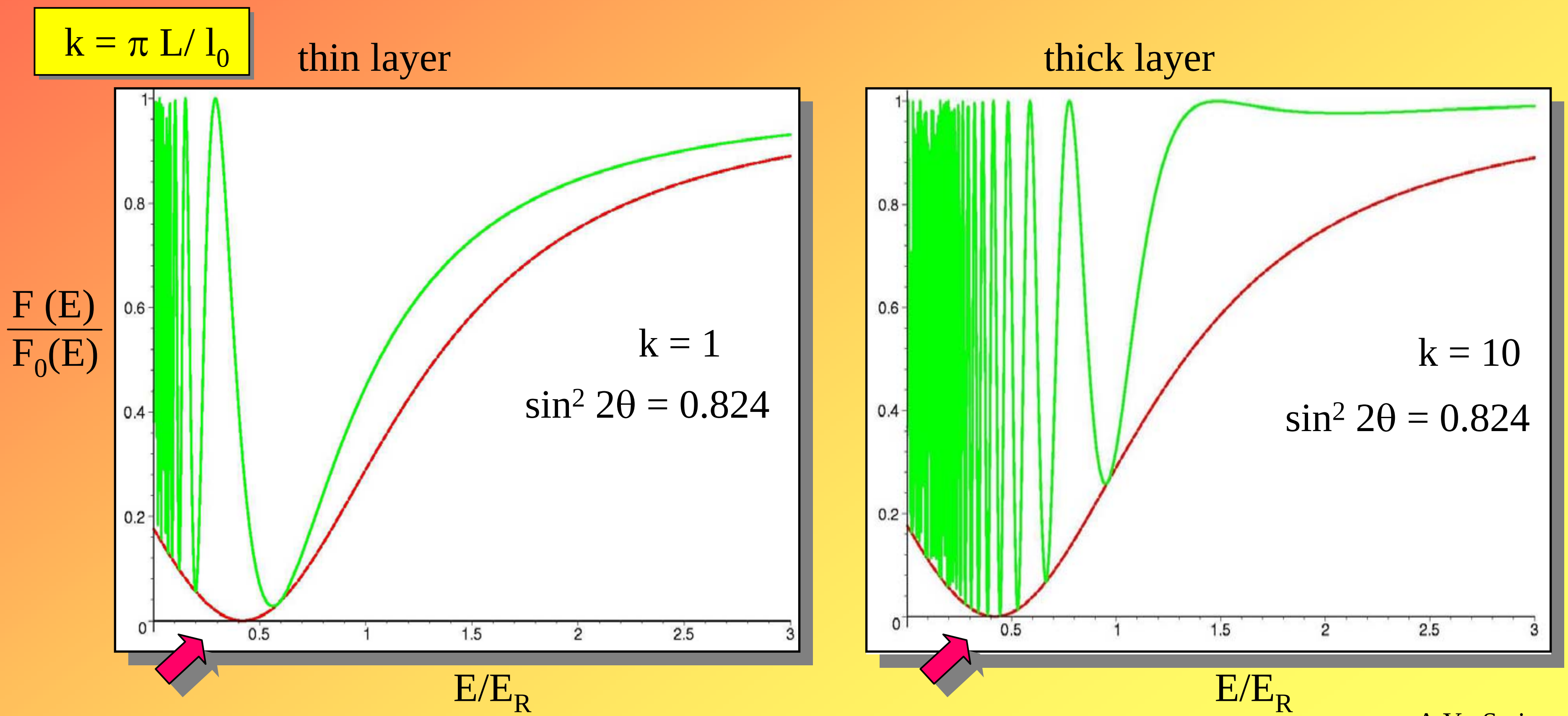
$\Delta\phi_m = (H_2 - H_1) L$

Parameters of oscillations (depth and length)
are determined by mixing in matter
and by effective energy split in matter

$\sin^2 2\theta, \; l_\nu$ → $\sin^2 2\theta_m, \; l_m$

Resonance enhancement of oscillations





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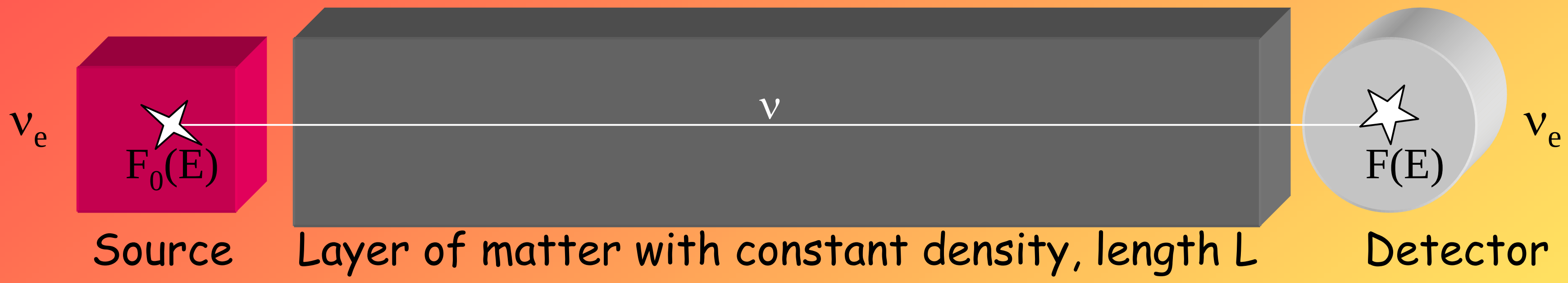
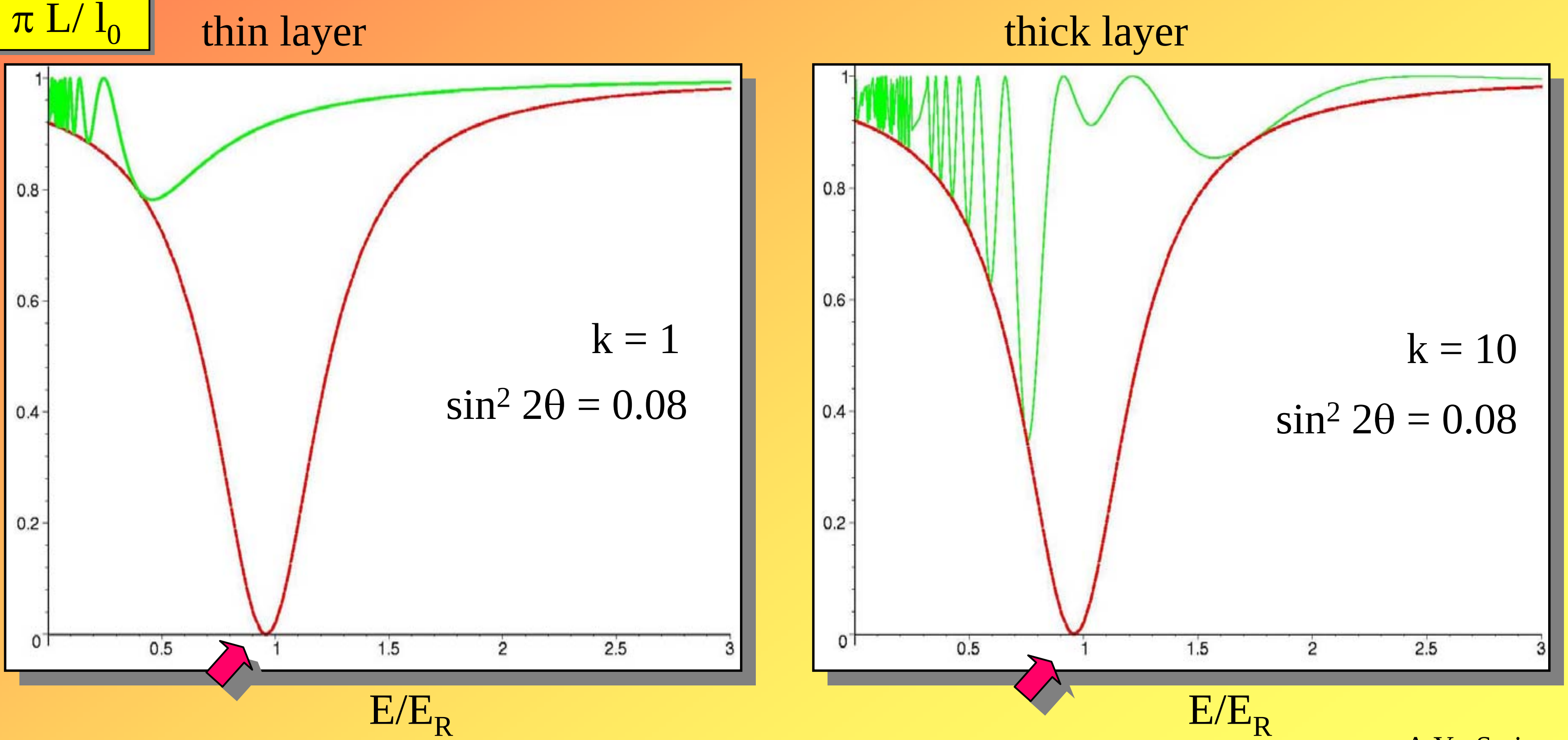
Resonance enhancement of oscillations



$k = \pi L / l_0$



$\frac{F(E)}{F_0(E)}$

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Resonance enhancement of oscillations

resonance layer

- Continuity:
  neutrino and antineutrino semiplanes
  normal and inverted hierarchy

- Oscillations (amplitude of oscillations) are enhanced in the resonance layer

$E = (E_R - \Delta E_R)$ -- $(E_R + \Delta E_R)$

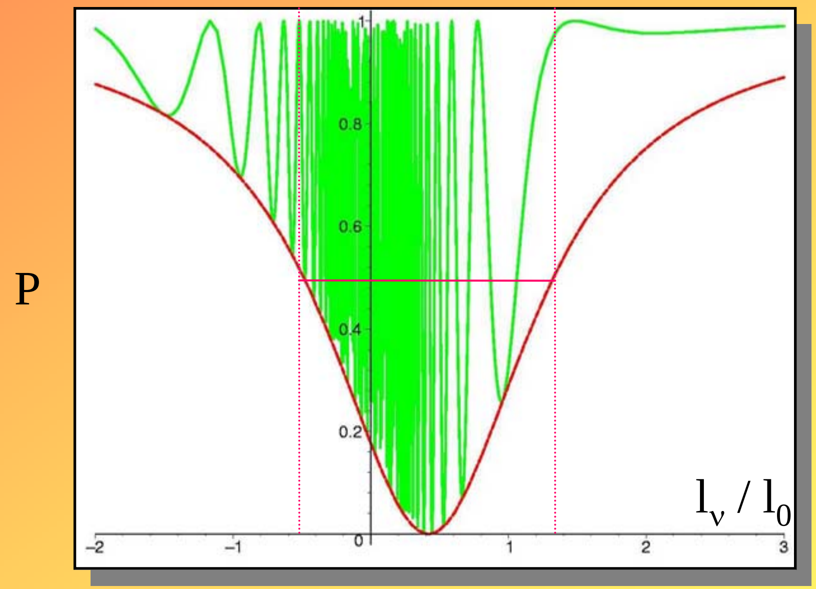
$\Delta E_R = E_R \tan 2\theta = E_R^0 \sin 2\theta$

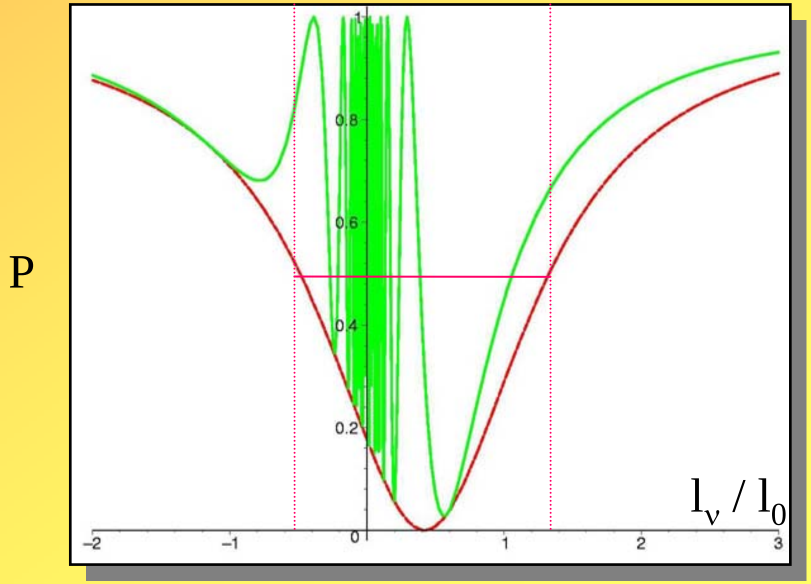
$E_R^0 = \Delta m^2 / 2V$

- With increase of mixing: $\theta \to \pi/4$

$E_R \to 0$

$\Delta E_R \to E_R^0$





MSW: adiabatic conversion

Non-uniform matter density changes on the way of neutrinos: $n_e = n_e(t)$

$H = H(t)$ depends on time

$\theta_m = \theta_m(n_e(t))$ mixing changes in the course of propagation

ν_{1m} ν_{2m} are no more the eigenstates of propagation $\to$ $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions

However

if the density changes slowly enough (adiabaticity condition) $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions can be neglected

- Flavors of eigenstates change according to the density change → determined by θ_m
- Admixtures of the eigenstates, ν_{1m} ν_{2m}, do not change → fixed by mixing in the production point
- Phase difference increases → according to the level split which changes with density

MSW

Effect is related to the change of flavors of the neutrino eigenstates in matter with varying density

Adiabaticity

- External conditions (density) change slowly so the system has time to adjust itself

$$\frac{\left|\frac{d\theta_m}{dt}\right|}{H_2 - H_1} << 1$$

Adiabaticity condition

➡ transitions between the neutrino eigenstates can be neglected

$$\nu_{1m} \not\leftrightarrow \nu_{2m}$$

➡ The eigenstates propagate independently

- Crucial in the resonance layer:
  - the mixing angle changes fast
  - level splitting is minimal

$$\Delta r_R > l_R$$

if vacuum mixing is small

$l_R = l_\nu / \sin 2\theta$ is the oscillation width in resonance

$\Delta r_R = n_R / (dn/dx)_R \tan 2\theta$ is the width of the resonance layer

- If vacuum mixing is large the point of maximal adiabaticity violation is shifted to larger dencities

$$n(a.v.) \rightarrow n_R^0 > n_R$$

$$n_R^0 = \Delta m^2 / 2\sqrt{2}\, G_F E$$

Adiabatic conversion and initial condition

The picture of conversion depends on how far from the resonance layer in the density scale the neutrino is produced

$n_0 > n_R$

$n_0 - n_R >> \Delta n_R$

Non-oscillatory conversion

$n_0 \sim n_R$

Interplay of conversion and oscillations

$n_0 < n_R$

$n_R - n_0 >> \Delta n_R$

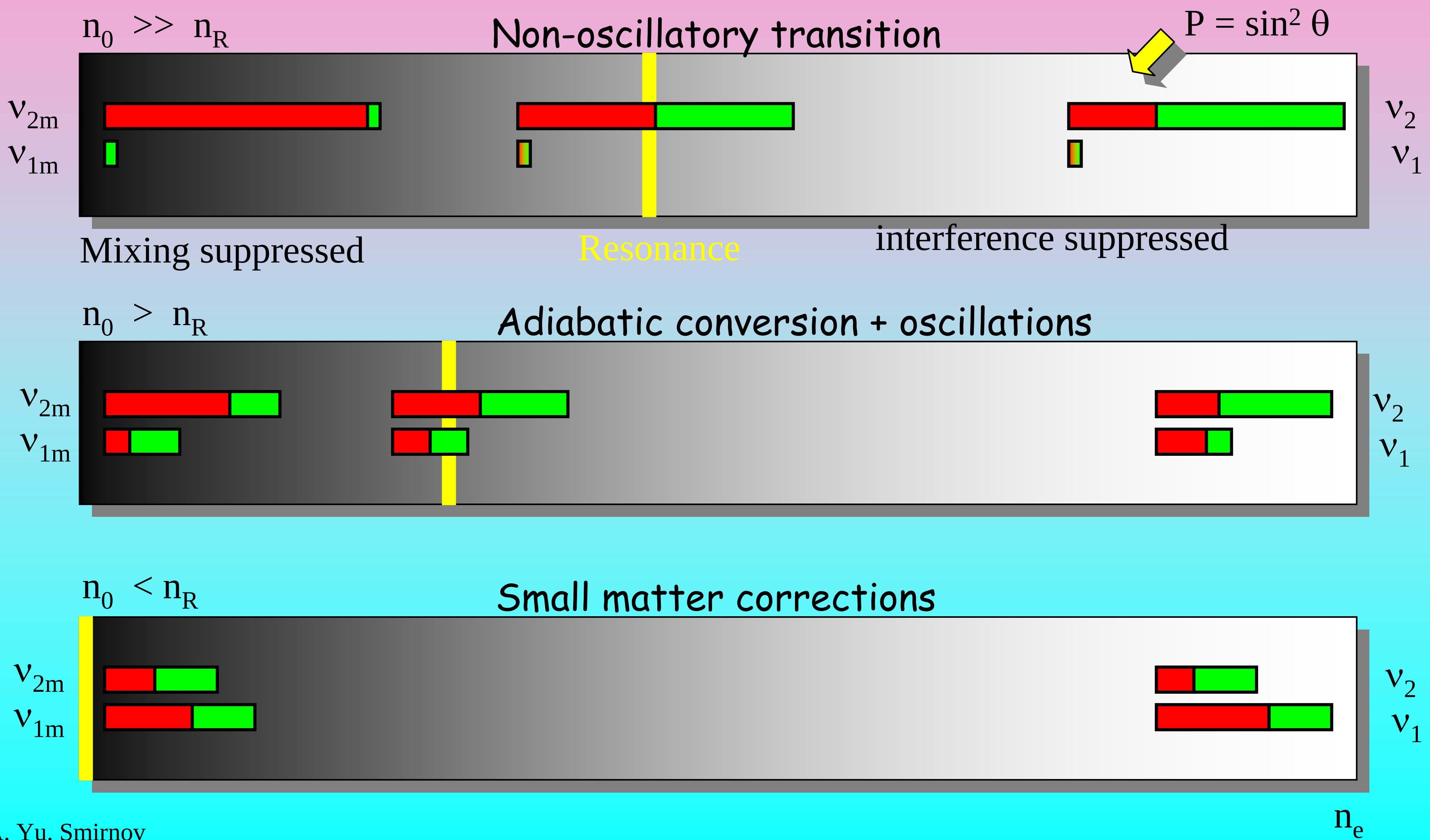
Oscillations with small matter effect

$n_R \sim 1/E$

All three possibilities are realized for the solar neutrinos in different energy ranges

Adiabatic conversion

ν_{1m} <-/-> ν_{2m}



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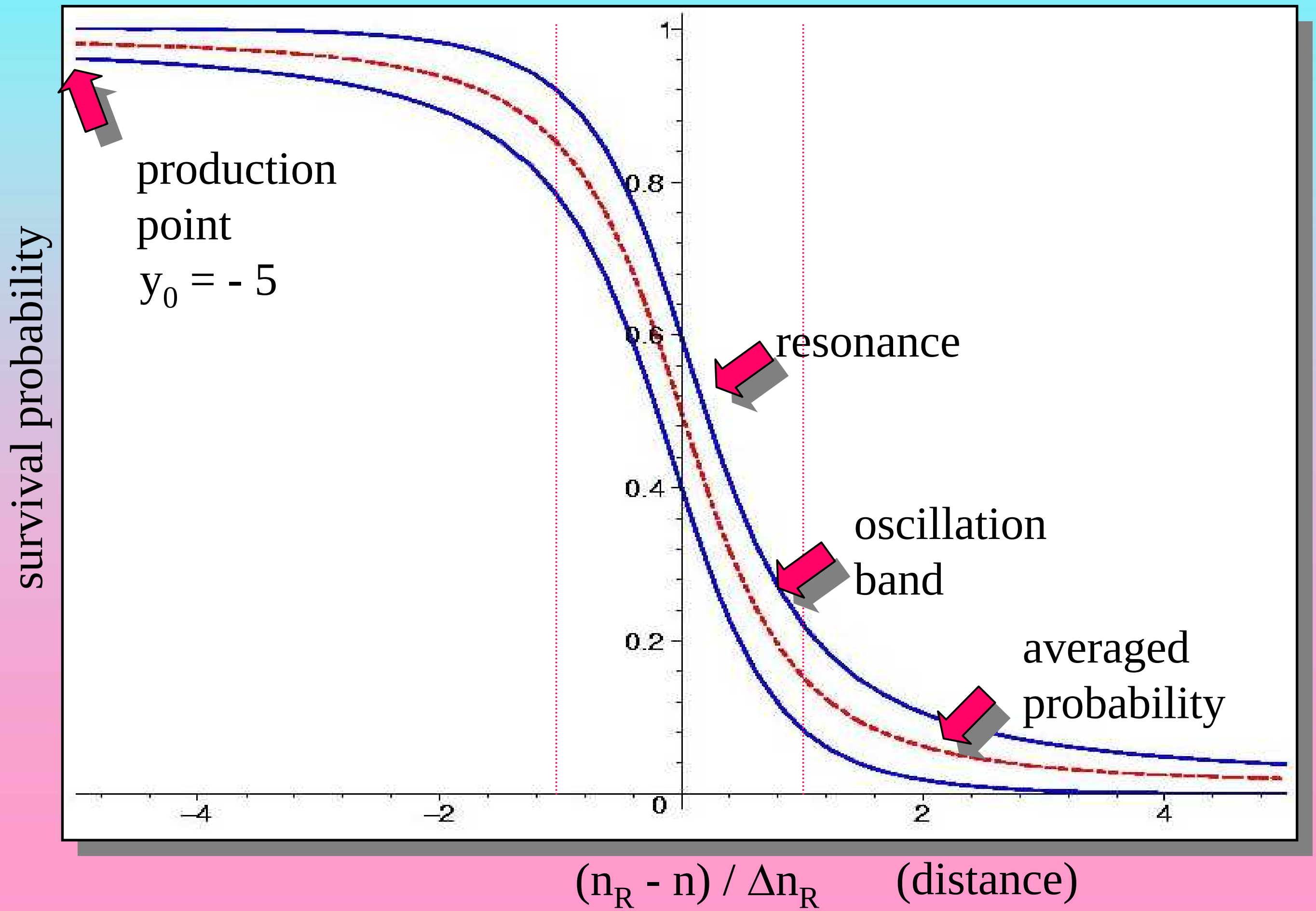
The MSW effect

The picture of adiabatic conversion is universal in terms of variable $y = (n_R - n) / \Delta n_R$
(no explicit dependence on oscillation parameters density distribution, etc.)
Only initial value y_0 matters.



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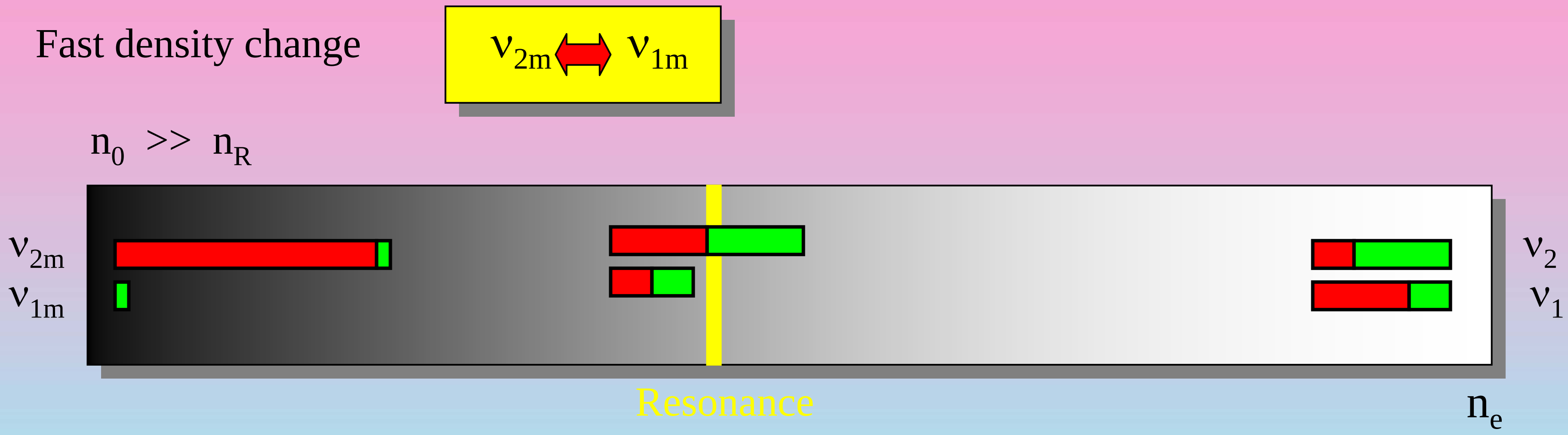
Adiabaticity violation

Fast density change



Admixture of ν_{1m} increases

Solar Neutrinos

$4p + 2e^- \Rightarrow {}^4He + 2\nu_e + 26.73$ MeV

electron neutrinos are produced

Adiabatic conversion in matter of the Sun

ρ : (150 $\Rightarrow$ 0) g/cc

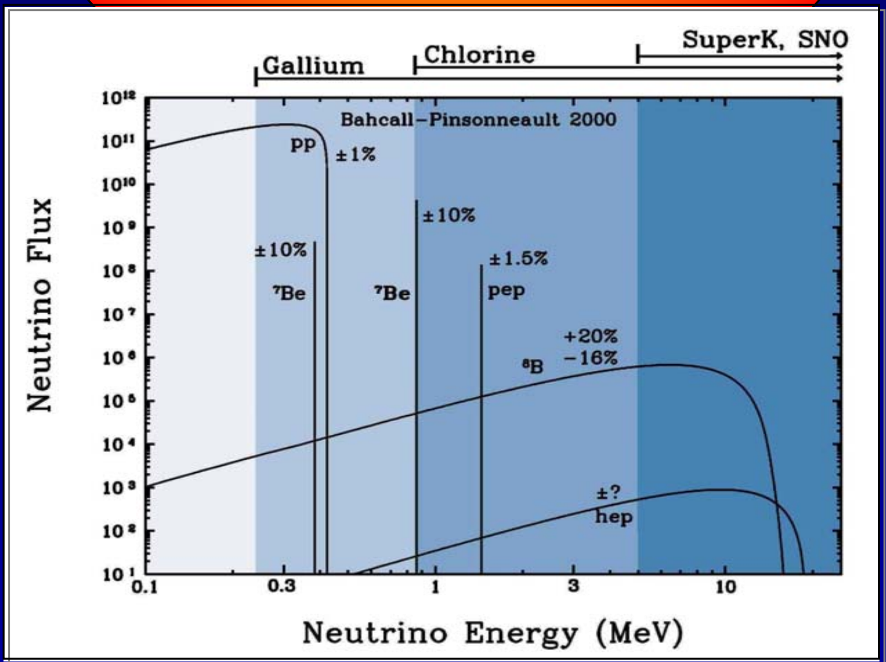
$F = 6\ 10^{10}\ cm^{-2}\ c^{-1}$

total flux at the Earth

Oscillations in vacuum

ν

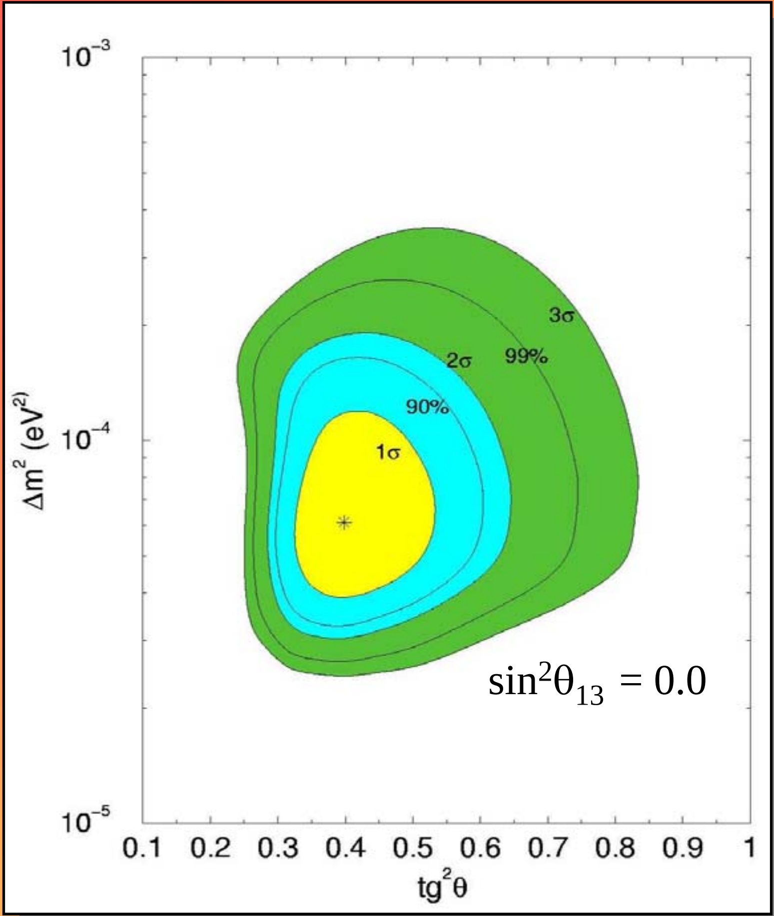
Oscillations in matter of the Earth



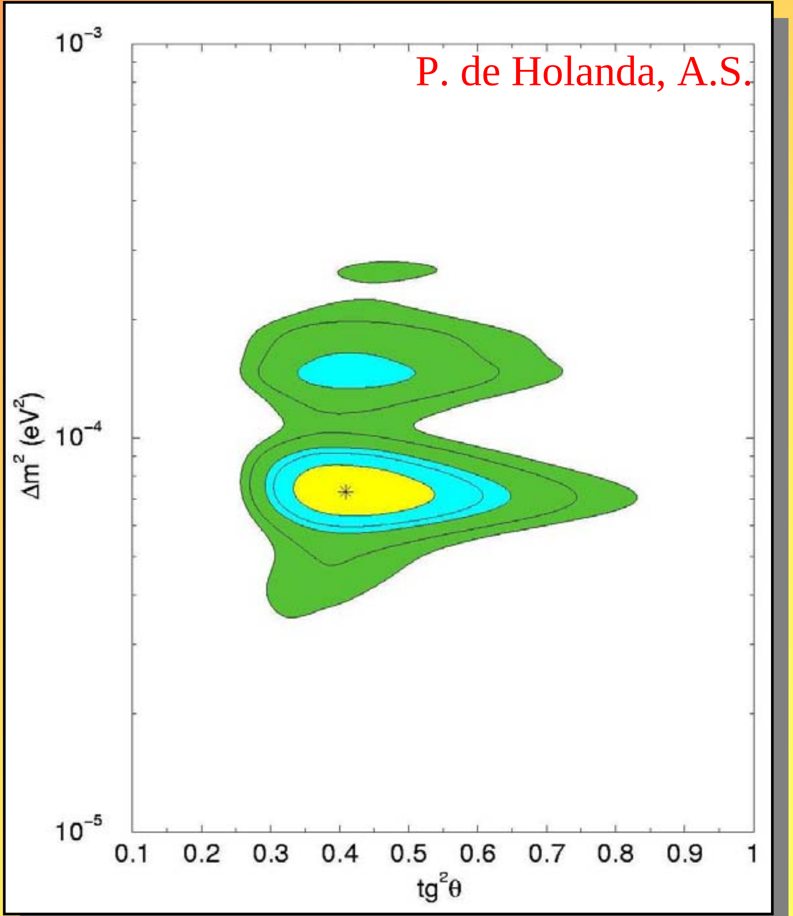
J.N. Bahcall

Large mixing MSW solution

solar data



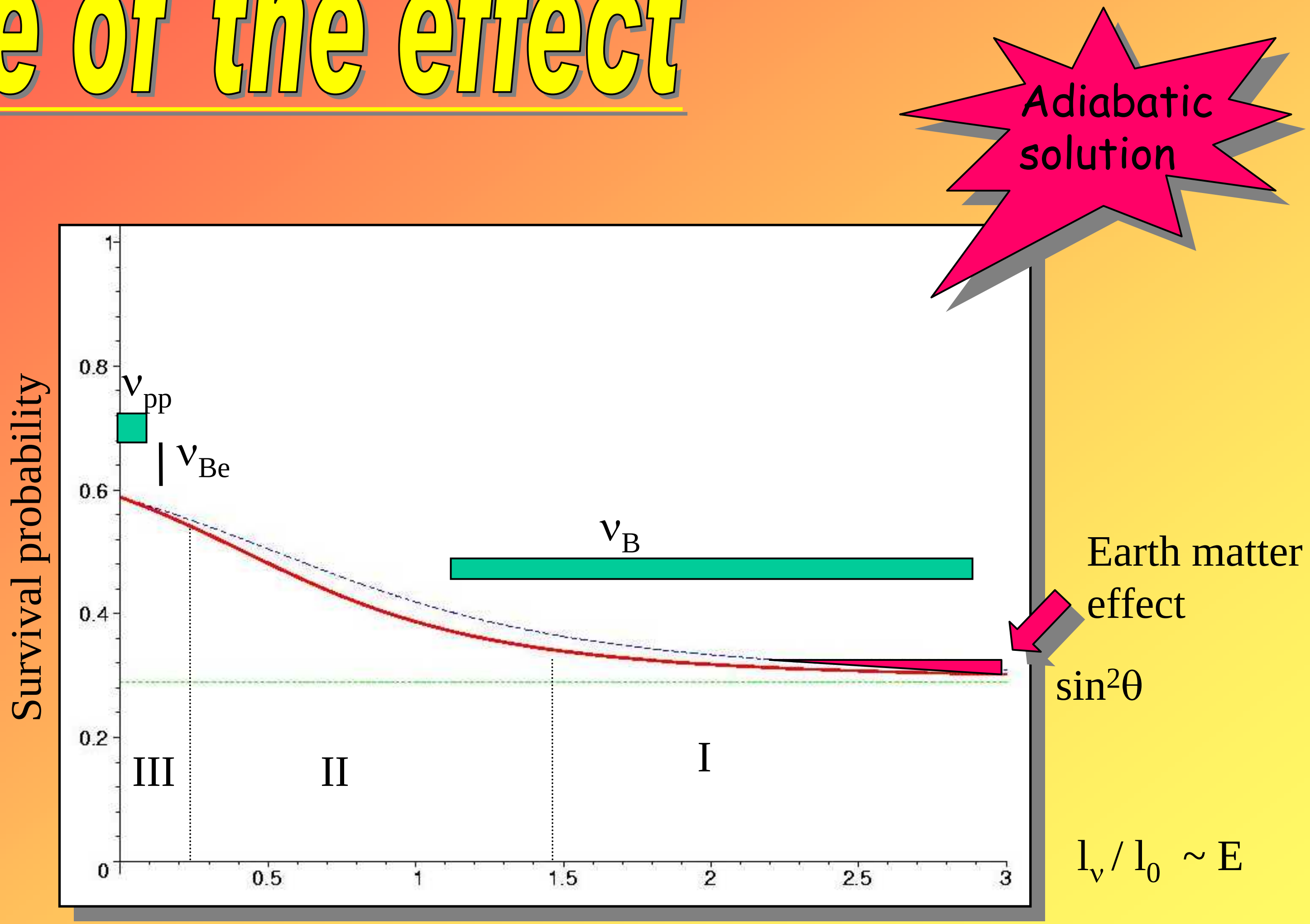
solar data + KamLAND



$\Delta m^2 = 6.8 \ 10^{-5}$ eV^2
$\tan^2\theta = 0.40$

$\Delta m^2 = 7.3 \ 10^{-4}$ eV^2
$\tan^2\theta = 0.41$

Profile of the effect



Oscillations with small matter effect

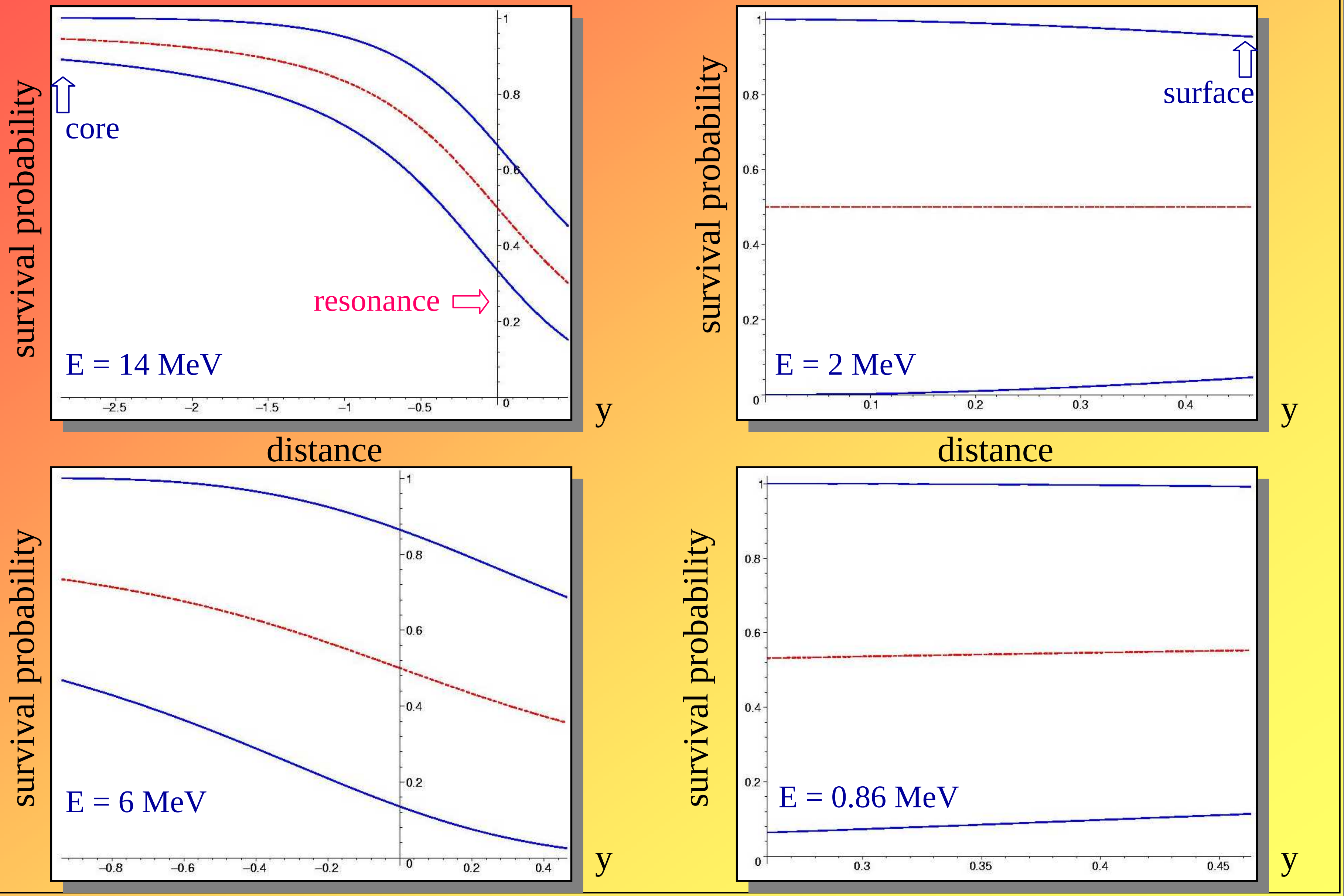
Conversion + oscillations

Conversion with small oscillation effect

Non-oscillatory transition

Conversion inside the Sun

$\tan^2\theta = 0.41$,
$\Delta m^2 = 7.3\ 10^{-5}\ eV^2$



LMA MSW solution

An example: E = 10 MeV

Resonance layer:

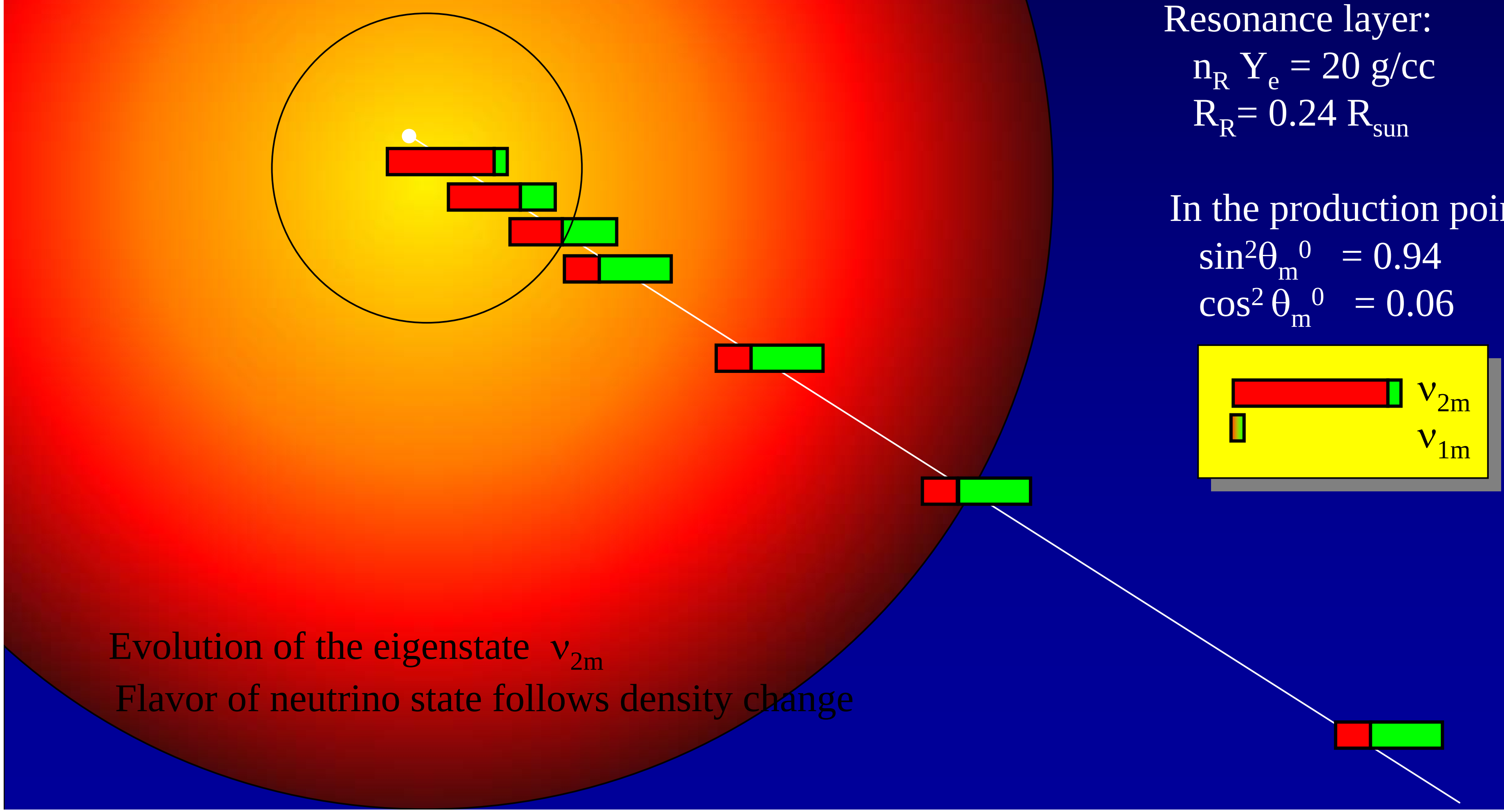
$n_R\ Y_e$ = 20 g/cc

R_R= 0.24 R_{sun}

In the production point:

$\sin^2\theta_m^0$ = 0.94

$\cos^2\theta_m^0$ = 0.06

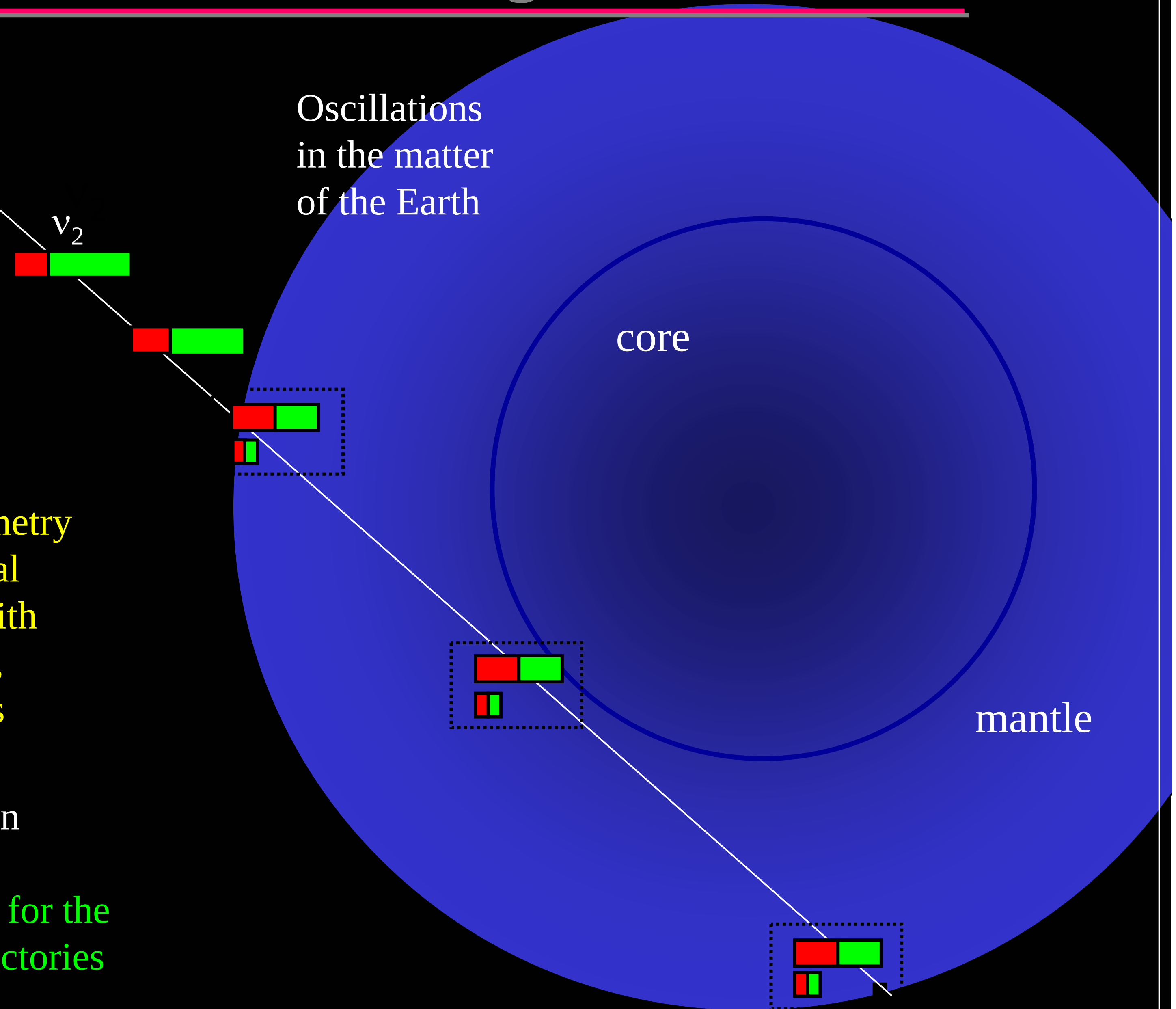


Evolution of the eigenstate ν_{2m}

Flavor of neutrino state follows density change

Inside the Earth. Regeneration

Regeneration of the ν_e flux



- Day - Night asymmetry
- Variations of signal during nights (zenith angle dependence),
- Seasonal variations

- Spectrum distortion

- Parametric effects for the core crossing trajectories

Inside the Earth

- Averaging of oscillations, divergency of the wave packets

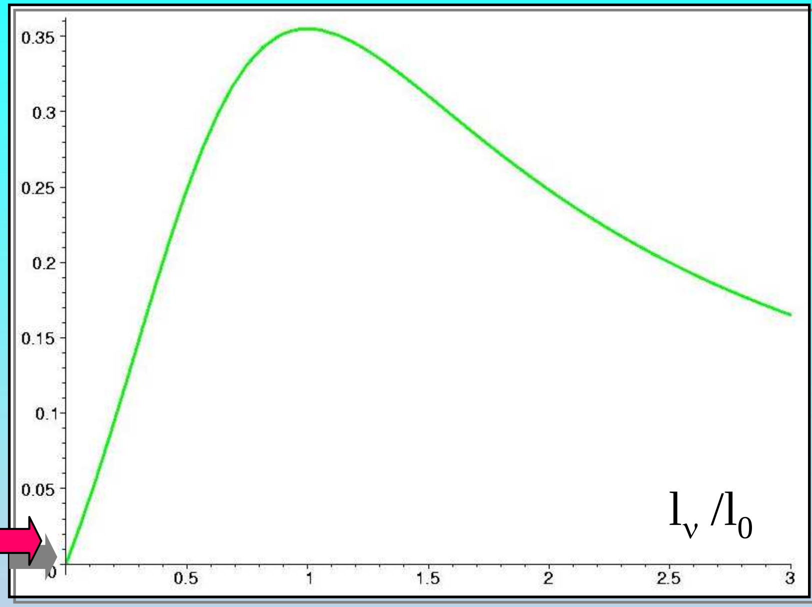
➡ incoherent fluxes of ν_1 and ν_2 arrive at the surface of the Earth

- ν_1 and ν_2 oscillate inside the Earth

➡ Regeneration of the ν_e flux

$l_\nu / l_0 \sim 0.03$
E = 10 MeV

f_{reg}



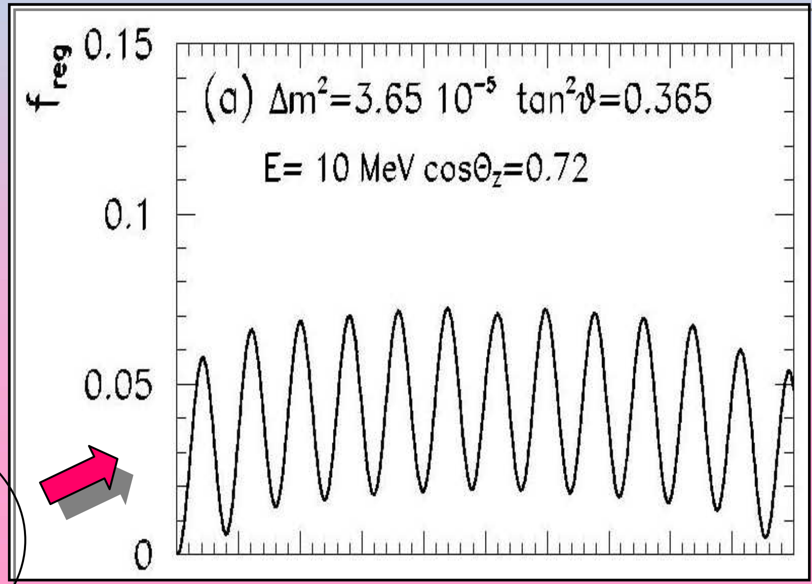
$P \sim \sin^2\theta + f_{reg}$

$f_{reg} \sim 0.5 \sin^2 2\theta \; l_\nu / l_0$

- The Day -Night asymmetry:

$A_{ND} = f_{reg}/P \sim 3 - 5\ \%$

Oscillations + adiabatic conversion



distance

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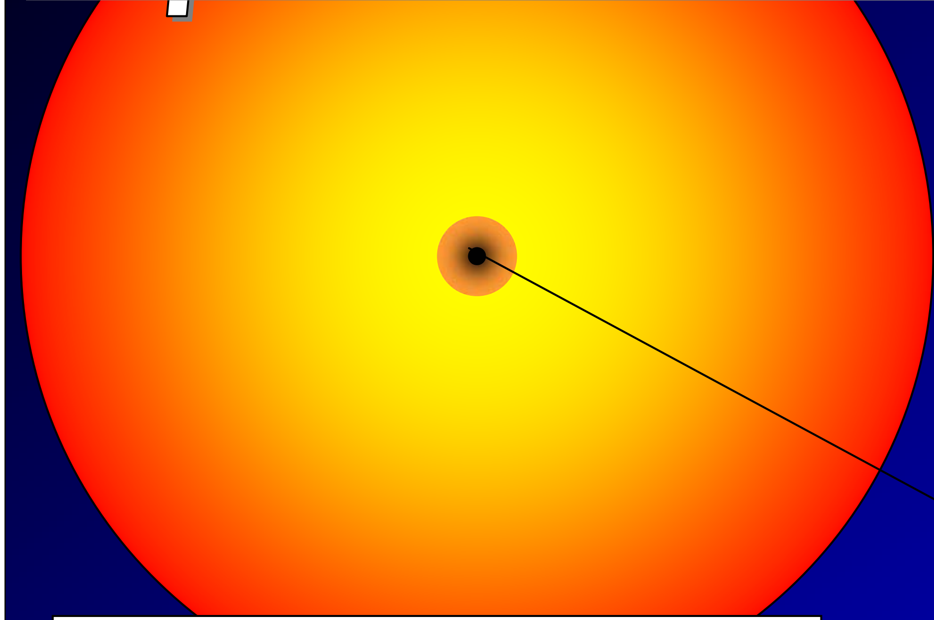
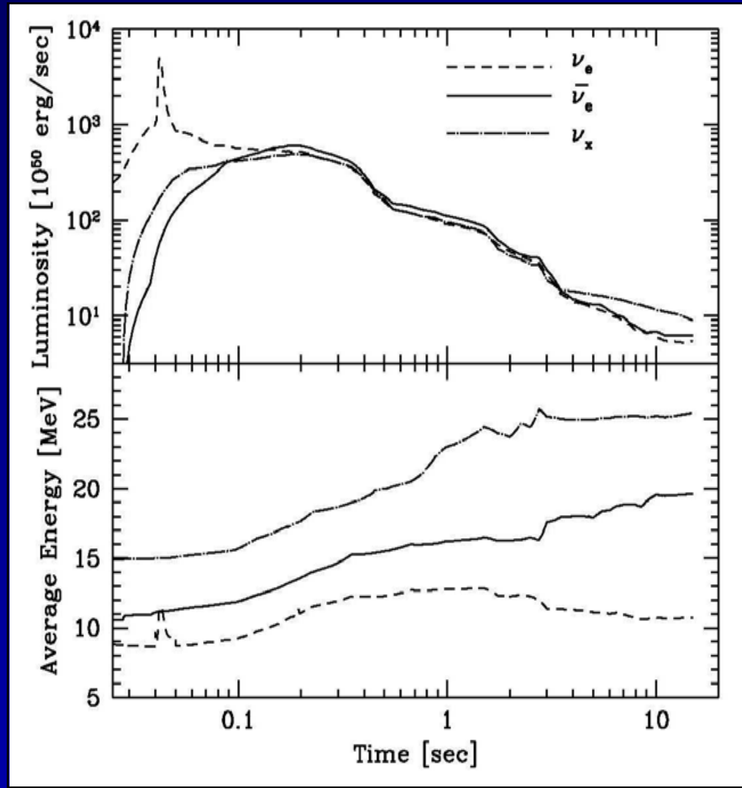
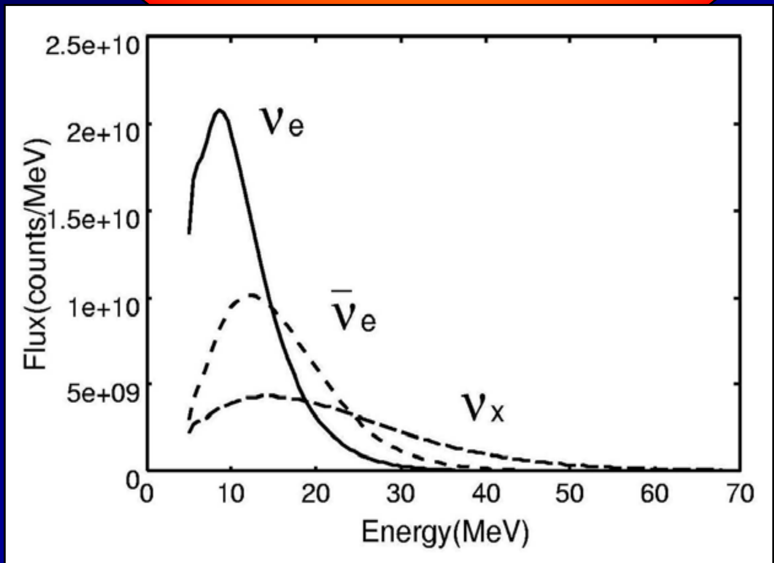
Supernova neutrinos

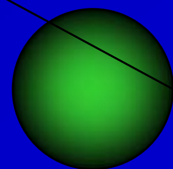






$\rho \sim (10^{11} - 10^{12})$ g/cc ⇨ 0

$E(\nu_e) < E(\bar{\nu}_e) < E(\nu_x)$



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SN neutrinos and MSW effect

- The MSW effect can be realized in very large interval of neutrino masses (Δm^2) and mixing

$\Delta m^2 = (10^{-6} - 10^{7}) \; eV^2$
$\sin^2 2\theta = (10^{-8} - 1)$

- Very sensitive way to search for new (sterile) neutrino states

- The conversion effects strongly depend on
  - Type of the mass hierarchy
  - Strength of the 1-3 mixing (s_{13})

A way to probe the hierarchy and value of s_{13}

- Small mixing angle realization of the MSW effect

If 1-3 mixing is not too small

$s_{13}^2 > 10^{-5}$

strong non-oscillatory conversion is driven by 1-3 mixing

In the case of normal mass hierarchy:

- $\nu_e \; \langle\!-\!\rangle \; \nu_\mu \; / \nu_\tau$ almost completely
- $F(\nu_e) = F^0(\nu_\mu)$ hard ν_e- spectrum
- No earth matter effect in ν_e - channel but in ν_e - channel
- Neutronization ν_e - peak disappears

SN87A and the Earth matter effect

- $F(\bar{\nu}_e) = F^0(\bar{\nu}_e) + p\,\Delta F^0$

  $p = (1 - P_{1e})$ is the permutation factor

  P_{1e} is the probability of $\nu_1 \to \nu_e$ transition inside the Earth

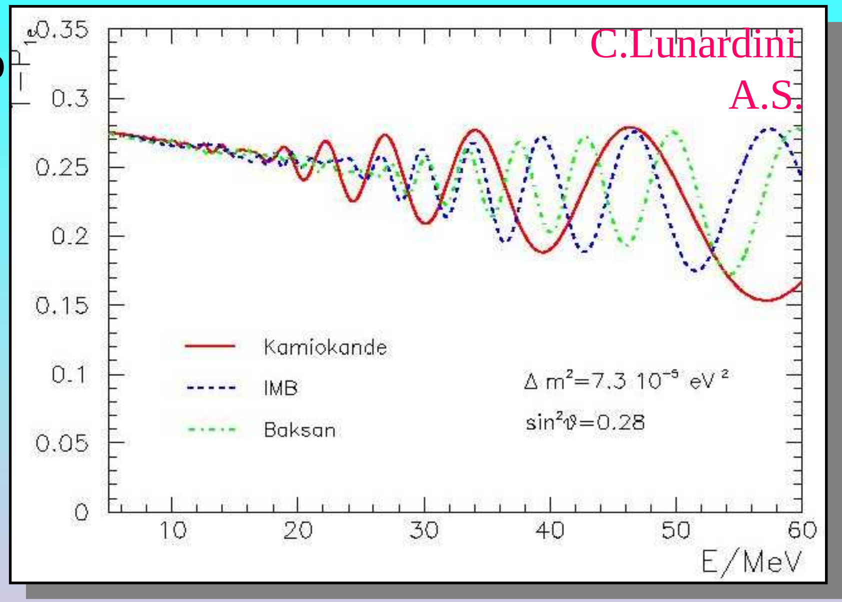
  $\Delta F^0 = F^0(\bar{\nu}_\mu) - F^0(\bar{\nu}_e)$

- p depends on distance traveled by neutrinos inside the earth to a given detector:

$$d = \begin{cases} 4363 \text{ km} & \text{Kamioka} \\ 8535 \text{ km} & \text{IMB} \\ 10449 \text{ km} & \text{Baksan} \end{cases}$$

- Can partially explain the difference of energy distributions of events detected by Kamiokande and IMB: at E ~ 40 MeV the signal is suppressed at Kamikande and enhanced at IMB

p



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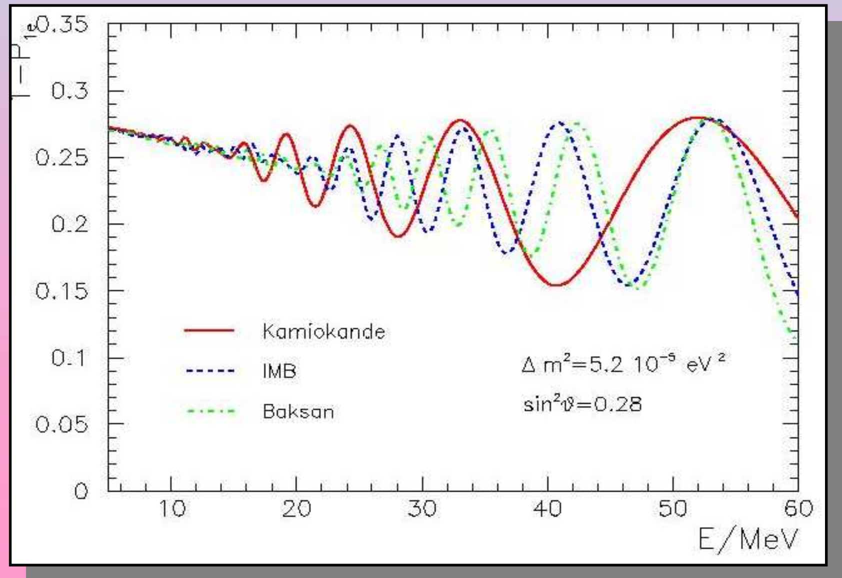
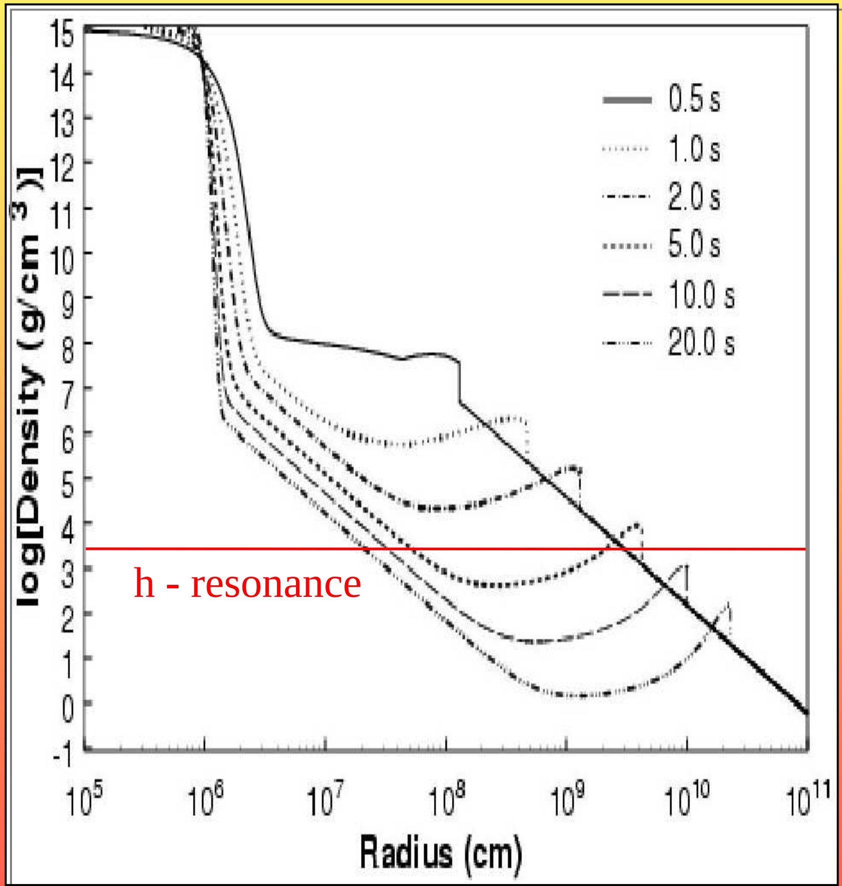


Shock Wave Effect

R.C. Schirato, G.M. Fuller, astro-ph/0205390



Density profile with shock wave propagation
at various times post-bounce

The shock wave can reach the region relevant for the neutrino conversion

$\rho \sim 10^4$ g/cc

During 3 - 5 s from the beginning of the burst

Influences neutrino conversion if $\sin^2\theta_{13} > 10^{-5}$

The effects are in the neutrino (antineutrino) for normal (inverted) hierarchy:

- change the number of events
  R.C. Schirato, G.M. Fuller, astro-ph/0205390
- ``wave of softening of spectrum''
  K. Takahashi et al, astro-ph/0212195
- delayed Earth matter effect
  C.Lunardini, A.S., hep-ph/0302033

Monitoring shock wave with neutrinos

G. Fuller

Studying effects of the shock wave
on the properties of neutrino burst
one can get (in principle) information on

- time of propagation
- velocity of propagation
- shock wave revival time
- density gradient in the front
- size of the front

Can shed some light on
mechanism of explosion

Summary

■ Two matter effects:

I. Resonance enhancement of oscillation in matter with constant density

2. Adiabatic (quasi-adiabatic) conversion in medium with varying density (MSW)

(a number of other matter effects exist)

■ Resonance enhancement of oscillations:

Can be realized for neutrinos propagating in the matter of the Earth (atmospheric neutrinos, accelerator LBL experiments, SN neutrinos ...)

■ Large mixing MSW effect:

Provides the solution of the solar neutrino problem

Determination of oscillation parameters

Δm_{12}^2 θ_{12}

■ Small mixing MSW effect:

Can be realized in supernova for 1-3 mixing

probe of 1-3 mixing, type of mass hierarchy

astrophysics, monitoring of a shock wave